

Midterm 4

Math 351 – Spring 2020

April 15th, 2020

Instructions

- *Write neatly and legibly!*
- Your camera *must* be on at *all times* and showing you properly. (You can only leave Zoom when you are done!)
- Leave the sound on (not the mic), so that you can *hear* incoming private messages or if I need to say something to all.
- You do not need to copy the statements. Just number your answers.
- Each problem must be solved in a different page, but items of the same problem can be in the same page.
- If you have any questions, send me a private message through the chat.
- You can only use your computer to look at the exam or to send me a message.
- **When you are done with the exam and are about to start scanning/uploading, send me a private message!** (Something like “*Scanning now.*”)
- Make sure your scans are legible before uploading them to Canvas.
- **When you are done uploading your solutions, send me a private message.** (Something like “*Done.*” No need for the time.) You can then leave Zoom.
- **Be prepared to, upon request (via private message), show me your surroundings!**

1) [25 points] Let $f = x^5 + x^4 + x^2 + 2x + 1$ and $g = x^4 + 2x$ in $\mathbb{F}_3[x]$. Find $\gcd(f, g)$ [in $\mathbb{F}_3[x]$]. [No need to express the GCD as a linear combination.]

2) [25 points] Let R be a domain. Prove that $R[x]$ is never a field.
[This was a HW problem.]

3) [25 points] Let F be a field and $f, g \in F[x]$, with $\deg(f) = \deg(g) > 0$, both *monic*. Prove that if $f \mid g$, then $f = g$.

4) [25 points] Let F be a field, $f \in F[x]$ and assume f is irreducible. Prove that for any $a \in F^\times$, the polynomial $a \cdot f$ is also irreducible.