April 6th, 2022

Math 351

Luís Finotti
Spring 2022

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 3

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) True of False? [If true, give a proof. If false, give a counter-example.] Remember that $U(R) = R^{\times} =$ set of units of R.
[**Note:** This was a HW problem.]

(a) [10 points] If R is an infinite commutative ring, then $U(R)$ is also infinite.

(b) [15 points] If S is a subring of the commutative ring R, then $U(S) = U(R) \cap S$.

2) Let R be a domain and $F = \mathrm{Frac}(R)$.

[**Note:** Both were done in class!]

(a) [10 points] Prove that for all $a, b, c \in R$, with $b, c \neq 0$, we have that $\dfrac{a}{b} = \dfrac{ac}{bc}$.

(b) [15 points] Prove that for all $a, b, c \in R$, with $c \neq 0$, we have that $\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$.

3) Answer the following questions. [*No justification needed.* But you *can* give justification, which can help with partial credit.]

(a) [5 points] The units of $\mathbb{Z}/8\mathbb{Z}$ are:

(b) [5 points] The prime field of $\mathbb{C}$ is:

(c) [5 points] The prime field of $\mathbb{F}_{11}$ is:

(d) [5 points] The field of fractions of $\mathbb{R}$ is:

(e) [5 points] The characteristic of $\mathbb{Z}/12\mathbb{Z}$ is:

4) [25 points] Let R be a commutative ring. Prove that if $a \in R$, then $(-1) \cdot a = -a$ *using only the commutative ring axioms* [provided in the last page] *and* the fact that $0 \cdot a = 0$ for all $a \in R$.

[**Hints:** This was a done in class! Note that it suffices to show that $a + ((-1) \cdot a) = 0$.]

Scratch:

Commutative Ring Axioms: A [non-empty] set with two operations, $+$ and $\cdot$, is a commutative ring if:

0. For all $a, b \in R$ we have that $a + b \in R$ and $a \cdot b \in R$.

1. For all $a, b \in R$ we have that $a + b = b + a$.

2. For all $a, b, c \in R$ we have that $(a + b) + c = a + (b + c)$.

3. There exists $0 \in R$ such that for all $a \in R$ we have $a + 0 = a$.

4. For all $a \in R$ there exists $-a \in R$ such that $a + (-a) = 0$.

5. For all $a, b \in R$ we have that $a \cdot b = b \cdot a$.

6. For all $a, b, c \in R$ we have that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

7. There is $1 \in R$ such that for all $a \in R$ we have that $1 \cdot a = a$

8. For all $a, b, c \in R$ we have that $a \cdot (b + c) = a \cdot b + a \cdot c$