April 27th, 2022

Math 351

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Spring 2022

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 4

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 6 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) Let

$$f = 3x^5 + 4x^4 + x^3 + x^2 + 3x + 2,$$
$$g = 4x^3 + 2,$$

be polynomials in $\mathbb{F}_5[x]$. Find $\gcd(f, g)$.

[**Note:** There is *no need* to express the GCD as linear combination of f and g.]

2) Let R be a commutative ring. Prove that $R[x]$ is a domain if and only if R is a domain.

[**Note:** This was done in class.]

3) Let F be a field, $f \in F[x]$, and $a_1, a_2, \ldots, a_k \in F$ be distinct roots of f, i.e., $f(a_i) = 0$ for $i = 1, 2, \ldots, k$. Prove that there is $g \in F[x]$ such that

$$f = g \cdot (x - a_1)(x - a_2) \cdots (x - a_k).$$

[**Hint:** We have proved the case when $k = 1$ in class and you *can* use that.]

4) Let R be a commutative ring, $a \in R$, $f \in R[x]$, and suppose that $(x-a) \mid f$. Prove that $(x-a)^2 \mid f$ if and only if $(x-a) \mid f'$ [where f' is the derivative of f].

[**Note:** This was a HW problem. You can use the formulas for the derivative from Calculus and the *Basic Lemma* for polynomial rings.]

Scratch: