Midterm 1

This is a take-home exam: You cannot talk to *anyone* (except me) about *anything* on this exam and you can only look at *our* book (Velleman) and class notes. *No other reference is allowed*, including the Internet. Failing to follow these instructions will result in a zero for the exam. Moreover, I will report the incident to the university and do all in my power to get the maximal penalty for the infraction.

As you will have a lot of time, I expect the solutions to be well written and clearly explained.

If you need *clarifications on any statement*, please use the "Q&A (Math Related)" forum on Blackboard. *Questions related to content/math of the exam should be submitted by e-mail!* Please use your best judgment on what is appropriate to ask in the forum.

Due date: Your solutions must be uploaded on Blackboard by Tuesday 06/10 by 11:59pm. Please send as a PDF and make sure your scanned/typed exam is clear and legible.

1) [15 points] Write the negation of the statement below as a positive statement [i.e., no negation before quantifiers].

$$\forall \epsilon \left[\epsilon > 0 \to \exists N \left(\forall n \left(n \geq N \to \frac{1}{n} < \epsilon \right) \right) \right]$$

[The universe is $\mathbb{R}$ here.]
[You don't *have* to show work here, but skipping steps makes it harder to understand what you've done, so might lead to less partial credit if you got it wrong.]

2) [10 points] Analyze the logical structure of the following statement:

Everyone who has a musician friend knows Pink Floyd.

Careful: Don't make multiple statements into one. For instance, something like

$$P(x) = x \text{ is married to someone who speaks Spanish}$$

is not a simple statement, as it contains two statements: x is married and whoever he is married to speaks Spanish. [Look in the book for examples!]

3) [10 points] Let

$$L(x, y) = x \text{ loves } y.$$

Translate the following statement to [natural sounding] English.

$$\exists x \, [L(\text{Alice}, x) \wedge \forall y \, ([L(\text{Alice}, y) \wedge y \neq x] \rightarrow y = \text{Alice})]$$

[Try not to lose precision when making it sound natural!]

4) [10 points] Write the truth table for $P \vee Q \rightarrow Q \wedge \neg R$.

5) [25 points] Let $P \mid Q$ denote "P and Q are not both true".

(a) Write the truth table of $P \mid Q$.

(b) Find a formula [involving P and Q] using only $\wedge$, $\vee$ and $\neg$ operations logically equivalent to $P \mid Q$.

(c) Find a formula logically equivalent to $\neg P$ using only $\mid$ [and P]. [*Show* that your formula is indeed equivalent!]

(d) Find a formula for $P \wedge Q$ using only $\mid$ [and P and Q]. [*Show* that your formula is indeed equivalent!]

(e) Find a formula for $P \vee Q$ using only $\mid$ [and P and Q]. [*Show* that your formula is indeed equivalent!]

6) [15 points] Let A, B and C be sets and consider:

$$S_1 = (C \setminus A) \cap (A \cup B),$$
$$S_2 = (A \cap C) \cup ((B \cap C) \setminus A).$$

(a) Give the Venn Diagrams for S_1 and S_2. [Label them clearly!]

(b) If the Venn Diagrams above show that $S_1 = S_2$, then show it using logic. If the diagrams are different, give simple *concrete* examples of A, B and C for which the corresponding S_1 and S_2 are different. [*Show work!*]

7) [15 points] Let I be a non-empty set of indices and $\{A_i \mid i \in I\}$ and $\{B_i \mid i \in I\}$ be families of sets indexed by I. Prove or disprove:

$$\left(\bigcap_{i\in I} A_i\right) \cap \left(\bigcap_{i\in I} B_i\right) = \bigcap_{i\in I}(A_i \cup B_i).$$