Midterm

This is a take-home exam: You cannot talk to *anyone* (except me) about *anything* on this exam and you can only look at *our* book (Velleman), our class notes [on Sage Math Cloud] and our solutions. *No other reference is allowed*, including the Internet. Failing to follow these instructions will result in a zero for the exam. Moreover, I will report the incident to the university and do all in my power to get the maximal penalty for the infraction.

As you will have a lot of time, I expect the solutions to be well written and clearly explained.

If you need *clarifications on any statement*, please use Piazza (non-private). *Questions related to content/math of the exam or that should not be seen by all should be submitted as **private messages** in Piazza!* Please use your best judgment on what is appropriate to ask in the forum.

Due date: Your solutions must be uploaded on Blackboard by Sunday 06/26 by 11:59pm. Please send as a PDF and make sure your scanned/typed exam is clear and legible.

1) Fill in the [*incomplete*] truth-table below [read the statements carefully!]:

P	Q	R	$(P \wedge Q) \to R$	$Q \vee \neg R$	$[\neg((P \wedge Q) \to R)] \to (Q \vee \neg R)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

[**Note:** If you want to type this, I will provide this table in Sage Math Cloud for you to copy and paste it.]

2) Prove or disprove: $(A \cup B) \setminus C = A \cup (B \setminus C)$.

3) Analyze the logical structure of the following statement: *"There are* exactly *two other people besides Alice who are as smart as she is"*. [Be careful with the *"exactly"*!]
You may assume that the universe set is the set of all people, say P, so that you can write, say $\exists x(\ldots)$, instead of $\exists x \in P(\ldots)$, for "there is a person x such that…".

4) Rewrite the [nonsensical] statement below as a positive statement [so no negations before quantifiers or parentheses/brackets, but $\notin$ and $\neq$ *are* allowed]. Here the universe is $\mathbb{R}$ [so $\exists x(\ldots)$ means $\exists x \in \mathbb{R}(\ldots)$] and I is the interval $(0, 1)$.

$$\neg\left[\forall x\left[(x \in I \lor x > 10) \leftrightarrow (\exists y(x \cdot y = 1))\right]\right]$$

5) Let $\mathcal{F}$ be a family of sets and A be a set. Rewrite the statement

$$\bigcup \mathcal{F} \subseteq \bigcap \mathscr{P}(A)$$

without using $\subseteq$, $\nsubseteq$, $\mathscr{P}$, $\cup$, $\cap$, $\setminus$, $\{$, $\}$ or $\neg$. [You may use $\in$, $\notin$, $=$, $\neq$, $\land$, $\lor$, $\rightarrow$, $\forall$ and $\exists$, though.]

6) Let A and B be sets. Prove that $A \setminus (A \setminus B) = A \cap B$.

7) Let $\mathcal{F}$ and $\mathcal{G}$ be non-empty families of sets. Prove that $\bigcup \mathcal{F}$ and $\bigcup \mathcal{G}$ are disjoint iff for every $A \in \mathcal{F}$ and every $B \in \mathcal{G}$ we have that A and B are disjoint.

8) Let U be a non-empty set. Prove that for every $A \in \mathscr{P}(U)$, there is a *unique* $B \in \mathscr{P}(U)$ [this B may depend on the choice of A] such that for every $C \in \mathscr{P}(U)$ we have $C \setminus A = C \cap B$. [Don't let the $\mathscr{P}(U)$ intimidate you. U here is just "the universe", i.e., all sets in here are contained in this U.]