Exam 1

You must upload the solutions to this exam by 11:59pm on Saturday 07/22. Since this is a take home, I want all your solutions to be neat and well written.

You can look at class discussions on Cocalc and *our* book only! You *cannot* look at our videos, solutions posted by me or *any* other references (including the Internet) without my previous approval. Also, of course, you cannot discuss this with *anyone*!

You can use a computer only to check your answers, as **you need to show work in all questions**.

1) [15 points] Use the *Extended Euclidean Algorithm* to write the GCD of 186 and 69 as a linear combination of themselves. *Show the computations explicitly!* [**Hint:** You should get 3 for the GCD!]

2) [13 points] Compute the LCM of 186 and 69 [the same numbers above!].

3) [15 points] Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$, then $a \mid (b \cdot c)$. [This is as simple as it gets! Don't make it hard!]

4) [15 points] Find the remainder of the division of 674378^{584} when divided by 5. *Show your computations explicitly!*

5) [12 points] Let $a = 2^5 \cdot 3^2 \cdot 11^4 \cdot 13$ and $b = 3^2 \cdot 5 \cdot 11^3$.

(a) Compute the prime factorization of $\gcd(a, b)$.

(b) Compute the prime factorization of $\operatorname{lcm}(a, b)$.

6) [15 points] Give the set of all solutions of the system

$$\begin{aligned} x &\equiv 4 \pmod{15}, \\ x &\equiv 22 \pmod{33}. \end{aligned}$$

[**Hint:** The system *does* have solution(s)!]

7) [15 points] Prove that there are no integers x and y such that

$$x^2 + y^2 = 1{,}000{,}000{,}000{,}003.$$

[**Hint:** What happens modulo 4?]