Exam 1

You must upload the solutions to this exam (as a PDF file) on Canvas by 11:59pm on Sunday 06/17. Since this is a take home, I want all your solutions to be neat and well written.

You can look at *our* book only! You *cannot* look at our videos, solutions posted by me or *any* other references (including the Internet) without my previous approval. Also, of course, you cannot discuss this with *anyone*!

1) Fill in the [*incomplete*] truth-table below:

P	Q	R	$P \wedge Q$	$Q \vee \neg R$	$(P \wedge Q) \to (Q \vee \neg R)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

2) Simplify the expression below using the formulas from pgs. 21 and 23 from the textbook:

$$\neg(\neg P \wedge Q) \vee [(P \wedge \neg R) \vee (Q \wedge \neg Q)]$$

[**Hint:** It should simplify to $\neg Q \vee P$.]

3) Show that the sets $(A \cup B) \setminus C$ and $(A \setminus B) \cup (B \setminus (A \cup C))$ are not equal [in general], by giving a *concrete* counterexample.
[**Note:** Drawing the Venn Diagrams is not enough! Although it would give you some partial credit and might help finding the counterexample.]

4) Express the [nonsensical] statement [with universe of discourse $\mathbb{R}$]

$$\neg\,[\exists x\ \ (\forall a \in \mathbb{Z}\ \ (a \leq x \to [\exists b \in \mathbb{Z}\ \ (a + b = x)]))]$$

as a positive statement.

5) Analyze the following statement: "Every parent has a child who eats only if no one is watching". You can only use the following statements:

$$P(x, y) = x \text{ is a parent of } y,$$
$$E(x) = x \text{ eats},$$
$$W(x, y) = x \text{ is watching } y.$$

You can leave implicit the universe of discourse as the set of all people.

6) Analyze the logical form of the following statements. [You may use $\in$, $\notin$, $=$, $\neq$, $\wedge$, $\vee$, $\to$, $\leftrightarrow$, $\forall$ and $\exists$, but not $\subseteq$, $\nsubseteq$, $\mathscr{P}$, $\cap$, $\cup$, $\setminus$, $\{$, $\}$ or $\neg$.]

(a) $x \in \bigcup \mathcal{F} \setminus \bigcap \mathcal{G}$, where $\mathcal{F}$ and $\mathcal{G}$ are families of sets;

(b) $X \in \bigcup_{i \in I} \mathscr{P}(A_i \cap B_i)$, where A_i and B_i, for $i \in I$, are indexed families.