Exam 2

You must upload the solutions to this exam (as a PDF file) on Canvas by 11:59pm on Sunday 06/24. Since this is a take home, I want all your solutions to be neat and well written.

You can look at *our* book only! You *cannot* look at our videos, solutions posted by me or *any* other references (including the Internet) without my previous approval. Also, of course, you cannot discuss this with *anyone*!

1) Prove that if $n \in \mathbb{R}$ and $n^2 \notin \mathbb{Z}$, then $n \notin \mathbb{Z}$.

2) Let $\mathcal{F}$ and $\mathcal{G}$ be non-empty families of sets. Prove that if $\mathcal{F} \subseteq \mathcal{G}$, then $\bigcup \mathcal{F} \subseteq \bigcup \mathcal{G}$.

3) Let A_i, for $i \in I$, be an indexed family of sets, with $I \neq \varnothing$. Prove that

$$\bigcap_{i \in I} A_i \in \bigcap_{i \in I} \mathscr{P}(A_i).$$

4) Let $\mathcal{F}$ and $\mathcal{G}$ be non-empty families of sets. Prove that $(\bigcup \mathcal{F}) \cap (\bigcup \mathcal{G}) = \varnothing$ if and only if for all $A \in \mathcal{F}$ and for all $B \in \mathcal{G}$ we have that $A \cap B = \varnothing$.

5) Let $\mathcal{F}$ be a non-empty family of sets. Prove that:

$$B \cup \left(\bigcup \mathcal{F} \right) = \bigcup \left(\mathcal{F} \cup \{B\} \right).$$

[**Note:** $\mathcal{F} \cup \{B\}$ is a family of sets that has all the sets of $\mathcal{F}$ and also B.]

6) Let $\mathcal{F}$ be a non-empty family of sets such that for any family of sets $\mathcal{G}$ such that $\mathcal{G} \subseteq \mathcal{F}$, we have that $\bigcup \mathcal{G} \in \mathcal{F}$. Prove that there exists a unique $A \in \mathcal{F}$ such that for any $B \in \mathcal{F}$, we have that $B \subseteq A$. [So, A contains every set of $\mathcal{F}$.]