MATH 142- EXAM IA-Feb. 1, 2005

Instructions. No credit for answers given without justification, even if correct (exception: see problem 4.) Calculators allowed, except for problem 3. Time given: 50 minutes.

1.[4,4] The left, right and trapezoidal rule approximations were used (with the same number n of intervals) to estimate $\int_0^2 f(x)dx$, where f is the function whose graph is shown. The estimates were 0.7811, 0.8675 and 0.9540.

(i)Which rule produced which estimate ? (Justify)

(ii) Between which two approximations does the true value of the integral lie? (Justify).

(Compare 5.9 2)

2.[4,4,4]Compute the following [no calculator!]:

(i)$\int_0^2 |x^2 - 1|dx$;

(ii)$\int_1^2 \frac{(x-1)^2}{x} dx$;

(iii)$\frac{d}{dx}\left(\int_0^{x^2} e^{t^2} dt\right)$.

(Compare 5.3 26, 34; 5.4 11)

3.[2,2,2,2](Justify each answer for credit.) Let $g(x) = \int_0^x f(t)dt$, where f is the continuous function in [0,10] whose graph is shown (note the areas -without signs- defined by the graph in each subinterval are given).)

(i) At what values of x does g have a local maximum ? (Ignore the endpoints.)

(ii) Where does g attain its absolute maximum value in [0,10]?

(iii) On what intervals is g concave upward?

(iv) At which points in (0,10) is g differentiable?

(Compare 5.4 20)

4.[4,4](i) Find approximations for the integral $\int_0^2 e^{-x^2} dx$ using (a) left-endpoint Riemann sums and (b) the midpoint rule (use 8 intervals in each case.) Give your answers rounded to 4 decimal places. For partial credit if the answer is wrong: include the points where the value of the function is computed.

(ii) How many intervals would you have to take (for each of the two methods) to guarantee the absolute error is smaller than 10^{-5}?

(Compare 5.9 16)