MATH 142- EXAM II-Feb.15, 2005

Instructions. No credit for answers given without justification, even if correct. Calculators allowed *except* for problem 4. Time given: 50 minutes.

1.[4,4](i) For the function $f(x)$ on [-2,1] whose graph is given below (concave down), the following information about Riemann (or trapezoidal) sums with $n = 50$ is known:

$$S_{50}^{left}[-2,0] = 7.253 \quad S_{50}^{right}[-2,0] = 7.413 \quad S_{50}^{midpt}[-2,0] = 7.4128$$

$$S_{50}^{left}[0,1] = 4.677 \quad S_{50}^{right}[0,1] = 4.657 \quad S_{50}^{trap}[0,1] = 4.667.$$

Use this to find a number that approximates $\int_{-2}^{1} f(x)dx$ as closely as possible, but is guaranteed to be *smaller* than the integral.

(ii) Given that $\max_{[-2,1]} |f'| \leq 5$ and $\max_{[-2,1]} |f''| \leq 4$, estimate the error involved in your approximation.

(Compare exam 1, problem 1)

2.[4,4]Let $f(x) = (1 + x^2)^{-1} - 1/2$ for $|x| \leq 1$, $f(x) = 0$ for $|x| > 1$ (graph of f given below.) Define

$$g(x) = \int_{-1}^{x} f(t)dt.$$

(i) Sketch the graph of $g(x)$ (for all $x \in \mathbb{R}$), indicating where it is increasing/decreasing/constant, concave up/concave down.

(ii) Find $\lim_{x\to+\infty} g(x)$.

(Compare exam 1, problem 3

3.[4,4,4,4] Compute the following indefinite integrals:

(i)$\int \sin^3 t dt$ (remember 3=2+1.)

(ii)$\int x^{-1/3}(1 - x^{2/3})^{3/2}dx$.

(iii)$\int \frac{x}{(x-1)(x-2)}dx$.

(iv)$\int \arctan x dx$.

(Compare text p.403 and practice handout)

4.[4] Sketch the region in the (x, y) plane bounded by the curves $x+y^2 = 2$ and $x + y = 0$ and compute its area.

(Compare 6.1 no. 12)