

MATH 142- EXAM 4- April 8, 2005

Instructions. No credit for answers given without justification. Calculators allowed. Time given: 60 minutes.

1. [3,3] For the region $D \subset \mathbb{R}^2$ given below, find (i) the y-coordinate of the center of mass and (ii) the volume of the solid obtained by rotating D about the x-axis (not necessarily in this order). You may use *Pappus' theorem*.

2. [4,4] Lengths of human pregnancies are normally distributed with mean 268 days and standard deviation 15 days. (i) What percentage of pregnancies lasts between 250 days and 280 days? (ii) What is the *median* duration of a human pregnancy? *You may use:* if N has the standard normal distribution (mean 0, std. dev. 1), probabilities for N are given below.

3. [3,3,3] Decide whether each of the following improper integrals converges or diverges (justify briefly; no need to compute the integral).

(i) $\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx;$

(ii) $\int_1^2 \frac{1}{(x^2-4)^{1/3}} dx;$

(iii) $\int_0^2 x^2 \ln x dx$ (*Hint:* set $u = \ln x$.)

4. [3,3,3] Decide whether each of the following series converges or diverges (justify briefly.) *Note:* you will only need to use what you know about geometric series, 'telescoping' series, and/or the 'divergence test'.

(i) $\sum_{n=1}^\infty \frac{(-6)^{n-1}}{5^{n-1}}.$

(ii) $\sum_{n=1}^\infty [\sin(\frac{1}{n}) - \sin(\frac{1}{n+1})].$

(iii) $\sum_{n=1}^\infty \frac{n}{n+5}.$