MATH 241- EXAM I-Feb. 8, 2005

Instructions. No credit for answers given without justification, even if correct. Calculators not allowed. Time given: 50 minutes.

1.[4, 4] (i) Find the vector component of $(-1, 3, 1)$ along the unit normal vector to the plane $x + y + z = 0$.

(ii) Find the decomposition $(-1, 3, 1) = \mathbf{v} + \mathbf{w}$, where $\mathbf{v}$ is a point in the plane in (i) and $\mathbf{w}$ is perpendicular to the plane (that is, find the vectors $\mathbf{v}$ and $\mathbf{w}$.)

(Compare 9.3 21, 25)

2. [4,4] (i) Find parametric equations for the line of intersection of the planes $x + y + z = 2$ and $2x - y + 3z = 1$.

(ii) Find an equation for the plane through $(-1, 2, 1)$, containing the line in (i). (*Hint:* take 2 arbitrary points on the line, reducing this to a plane-through-3 points problem.)

(*Compare 9.5 27)*

3. [4,4,4] A particle moves along the helix $\mathbf{r}(t) = (4\cos t, 3t, 4\sin t)$ with *constant* speed $v = |\mathbf{r}'| = 5$.

(i) Find the unit tangent vector $\mathbf{T}$ when $t = \pi/4$;

(ii) The acceleration vector $\mathbf{r}''(\pi/4)$ may be decomposed as:

$$\mathbf{r}''(\pi/4) = a_T\mathbf{T} + a_N\mathbf{N},$$

where **T, N** are the unit tangent and unit normal vectors at $t = \pi/4$. Compute the numbers a_T and a_N.

(iii) What is the curvature of the path when $t = \pi/4$?

Compare 10.3 11, 10.4 30

4.[4,4] (i) Write the surface with equation $x^2 + y^2 + 2y + z = -1$ in 'standard form' and identify it; sketch the cross-section defined by $z = -5$.

(ii) Write down an equation for the tangent plane to the surface at $(1, -1, -1)$. (*Hint:* write the surface as $z = -x^2 - y^2 - 2y - 1$.)

Compare 9.6 21, 11.4 1