MATH 241- EXAM 3-March 16, 2005

Instructions: Time given= 60 min; calculators allowed.

1.[4] Let $f(x,y) = x^2y^2$, $x(s,t) = e^s \cos t$, $y(s,t) = e^s \sin t$. Compute the gradient vector $\nabla u = (u_s, u_t)$ of the composition:

$$u(s,t) = f(x(s,t), y(s,t)).$$

2.[5,5,3,3] For the function of two variables

$$f(x,y) = 3x - x^3 - 2y^2 + y^4 :$$

(a) Find all the critical points of f;

(b) Classify the critical points (x,y) with $y > 0$ (as local maxima, local minima or saddle points);

(c) *Choose one* of the critical points found in (b) and write the second order Taylor approximation of $f(x,y)$ near that point;

(d) For *each* critical point in part (b), sketch the approximate diagram of level sets of f, in a neighborhood of that critical point.

3.[10] Find the absolute maximum and minimum values of the function of three variables:

$$f(x,y,z) = xy + z^2$$

under the constraint $x^2 + y^2 + z^2 = 4$.

4.[4,4] Consider the double integral:

$$\int_0^1 \int_{x^2}^1 x^2 \sin(y^3) dy dx.$$

(i) Sketch the region of integration;

(ii) Compute the value of the integral (change the order of integration, if necessary).