MATH 241- EXAM 4-April 8, 2005

Instructions: Time given= 50 min; calculators allowed.

For full credit, calculations must be performed to the end- i.e. your answer should be a real number, NOT a one-variable integral; you may use your calculator.

1.[4,4] For the double integral below: (i) sketch the region of integration, including the 'slicing' implicit in the given order of integration; (ii) reverse the order of integration and evaluate the integral.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}dxdy.$$

2.[9] Find the volume of the solid bounded by the cylinder $x^2+y^2=1$ and the planes $x+z=1$, $x=0$, $z=-1$ (sketch given). *Hint:* set up a double integral for the volume and compute it in polar coordinates.

3.[9] Find the z coordinate of the center of mass of a solid hemisphere of radius R ($x^2+y^2+z^2 \leq R^2, z>0$) if the mass density at any point is given by its distance z to the base. Given: total mass $M=(\pi/6)R^4$. (*Hint:* this is easier in spherical coordinates (ρ,θ,ϕ), with $z=\rho\cos\phi$).

4.[6,6] Consider the vector field in $\mathbb{R}^2$:

$$\mathbf{F}(x,y)=(4xy, 2x^2+y).$$

(i) Show that $\mathbf{F}$ is conservative, and find a potential function for $\mathbf{F}$.

(ii) Let C be the arc of the parabola $y=x^2$ from the origin to (2,4). Find the value of the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$. (Use either the definition of line integral or the answer of part (i)).