NEWTON'S *Principia: Structure and a selection of results*

AXIOMS, OR THE LAWS OF MOTION

Law 1: *Every body perseveres in its state of being at rest or moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.*

Law 2: *A change in motion is proportional to the motive force impressed ad takes place along the straight line in which that force is impressed.*

Law 3: *To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction.*

BOOK 1: THE MOTION OF BODIES (Sections 1-14)

SECTION 1: MATHEMATICAL LEMMAS
SECTION 2: TO FIND CENTRIPETAL FORCES (Prop. 1-10)

Prop. 1. The areas which bodies made to move in orbits [under the action of a central force] describe by radii drawn to the center of forces lie in unmoving planes and are proportional to the times.

Prop. 2 [Conversely,] a body that moves in a plane so that radii drawn to a fixed point (or to a point describing uniform rectilinear motion) describe areas proportional to times is moving under the action of a central force.

Prop. 4. The centripetal forces acting on bodies undergoing uniform circular motion are proportional to the the square of the arcs described in a given time, divided by the radius of the circle.

Cor. 6-7. [Under uniform circular motion,] if the periodic times are as the 3/2 power of the radii, the centripetal forces will be inversely as the square of the radii, and conversely; in general, if the periodic time is as the n-th power of the radius, the centripetal force will vary inversely as the 2n-1 power of the radius.

Prop.6, Cor. 5. Given any curvilinear figure in a plane and a fixed point S, to which the centripetal force is always directed, it is possible to find the law of centripetal force that will keep a body moving along that curve.

Prop. 10. If a body revolves in an ellipse, the law of centripetal force toward the *center* of the ellipse is: directly proportional to the distance from the body to the center.

Scholium: if the center goes off to infinity, so that the ellipse turns into a parabola, the force, now tending to an infinitely distant center, turns out to be uniform (Galileo's theorem.)

SECTION 3: THE MOTION OF BODIES IN ECCENTRIC CONIC SECTIONS (Prop. 11-17)

Prop. 11-12-13: let a body revolve in an ellipse; the law of centripetal force toward a *focus* of the ellipse is: inversely as the square of the distance from the body to the focus. (Similarly for a hyperbola or parabola.)

Cor. 1: If a body is acted upon by a centripetal force of magnitude inversely proportional to the square of distance to the center of force, the body will move on a conic section with a focus at the center of force.

Prop. 15: If a body moves on an ellipse under the action of a centripetal force inversely proportional to the square of the distance to the center, squares of periodic times are proportional to cubes of the major axis.

Prop. 17 [Problem:] If a body moves on a conic under a centripetal inverse-square law with center at a focus, to find the conic given the initial position and velocity
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SECTIONS 4-5: MATHEMATICAL PROPOSITIONS ON CONIC SECTIONS (Prop. 18-29)
SECTION 6: TO FIND MOTIONS IN GIVEN ORBITS [KEPLER'S PROBLEM] (Props. 30-31)

Prop 30-31: If a body moves in a parabolic (or elliptic) trajectory, to find its position at a given time.

Lemma 28: *No oval figure exists whose area, cut off by straight lines at will, can in general be found by means of equations finite in the number of terms and dimensions.*

SECTION 7: RECTILINEAR ASCENT AND DESCENT OF BODIES (Prop. 32-39)

Prop. 32: Given a centripetal inverse-square force, to determine the spaces which a body falling straight down describes in given times.

SECTION 8:ORBITS OF BODIES MOVING UNDER ARBITRARY CENTRIPETAL FORCES (Prp.40-42)

Prop. 41: Supposing a centripetal force of any kind, and granting the quadrature of curvilinear figures, it is required to find the trajectories in which bodies will move, and also the times of their motions in the trajectories so found.

SECTION 9: motion along an orbit that is revolving about the center of forces (Prop 43-45)-apsides.
SECTION 10: motion of bodies of planes, under a force whose center is not on the plane; cycloids; pendulums; and motion on curved surfaces (Prop. 46-56)

Prop. 51: If a centripetal force to the center of a globe is as the distance to the center; and if, under the action of this force alone, the body oscillates in the perimeter of a cycloid; then the times of the oscillations will be equal, however unequal their amplitudes may be.

Prop. 56: Granting the quadrature of curvilinear figures, and both the law of centripetal force toward a given center and a curved surface whose axis passes through that center, to find the trajectory a body will describe on that surface, given its initial position and velocity.

SECTION 11: TWO- AND THREE- BODY PROBLEMS, AND PERTURBATIONS (Prop. 57-69)

Prop. 61: If two bodies, attracting each other with any kind of forces and otherwise not acted or impeded, move in any way whatever, their motions will be the same as if they were not attracting each other, but were each being attracted with the same forces by a third body set in their common center of gravity; and the attracting forces will be the same with respect to the distance from the bodies to the common center, and with respect to the total distance between the bodies.

Prop. 66: Let three bodies mutually attract each other under an inverse-square law, with the two lesser ones (E) and (M) revolving about the greatest one, (S). Then if the greatest body (S) is moved by this action, the smaller inner body (M) will describe about the larger inner body (E), by radii drawn to it, areas more nearly proportional to the times, and a figure more closely resembling an ellipse, than would be the case if the greatest body were not attracted by the smaller ones, or were at rest.

(22 corollaries follow)

SECTION 12: ATTRACTIVE FORCES OF SPHERICAL BODIES (Prop. 70-84)

Prop. 70-71: if each point of a spherical surface is the center of a centripetal inverse-square law, a body inside the surface will not be attracted in any direction; a body outside the surface will be attracted as if the whole mass of the sphere were concentrated at its center.

Prop. 73-74: A body inside a homogeneous sphere, each of whose points is the center of a centripetal inverse-square law, is attracted by a force directed to the center of the sphere, proportional to the distance to the center; while a body outside the sphere is attracted by a force inversely proportional to the square of distance to the center.

Prop. 76: If two spherical bodies have mass densities depending only on distance to the center, they attract each other by a force inversely proportional to the square of the distance between their centers.

Prop. 84: To find the force with which a body is attracted by a segment of a sphere, when it is located along the axis of the segment, beyond the center of the sphere.

SECTION 13: ATTRACTIVE FORCES OF NONSPHERICAL BODIES (Prop. 85-93)

Prop. 90: If each point of a circular lamina is the center of a centripetal force with magnitude any function of distance, to find the force on a body placed on the line perpendicular to the lamina, through its center.

Prop. 91: To find the attraction on a body placed in the axis of a rotationally symmetric solid, each of whose points is the center of a centripetal force decreasing as any function of distance.

Prop. 93: Consider a homogeneous solid in the shape of an infinite half –space bounded by a plane P, whose points attract with a force decreasing as the n-th. power of distance, where n>2. Then the whole solid will attract a body with a force decreasing as the n-3 power of the distance from the body to the plane P (regardless of which side of P the body is on.)

SECTION 14: MINIMALLY SMALL BODIES IN THE VICINITY OF A GREAT BODY (94-98)

Prop. 94-95: If two homogeneous media are separated from each other by a space terminated on two sides by parallel planes, and a body passing through this space is attracted or impelled perpendicularly toward either medium and is not acted on by any other force, and the attraction is the same at equal distances from each plane (taken on the same side); I say that the sine of the angle of incidence onto either plane will be to the sine of the angle of emergence from the other plane in a constant ratio. Moreover, the velocity of the body before incidence is to its velocity after emergence as the sine of the angle of emergence to the sine of the angle of incidence.

BOOK 3: THE SYSTEM OF THE WORLD

(1)A set of *rules* for proceeding in natural philosophy; (2) the `*phenomena*' on which the system of the world is to be based; (3) application of the mathematical results in Book 1 to explain the motion of *planets* and *satellites* as a result of the universal action of gravity (Prop. 1-21); (4) analysis of the motion of the *moon* and the origin of *tides* (Prop. 22-39); (5) the motion of *comets* (Prop 40 to 42).

Rule 1: *No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena [parsimony].*

Rule 2: *The causes assigned to natural effects of the same kind must be, so far as possible, the same.*

Rule 3: *Those qualities of bodies that cannot be increased or diminished, and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally.*

Rule 4: *Propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions.*

Phenomenon 1: the satellites of Jupiter, by radii drawn to the center of Jupiter, describe areas proportional to the times, and their periodic times are as the 3/2 power of their distances from the center.

Phenomenon 2: Likewise for the satellites of Saturn.

Phenomenon 3: The orbits of the five primary planets- Mercury, Venus, Mars, Jupiter and Saturn- encircle the sun.

Phenomenon 4: The periodic times of the five primary planets and of either the earth about the sun or the sun about the earth are as the 3/2 power of their distances to the sun.

Phenomenon 5: The primary planets, by radii drawn to the earth, describe areas in no way proportional to the times, but by radii drawn to the sun traverse areas proportional to the times.

Phenomenon 6: The moon, by a radius drawn to the center of the earth, describes areas proportional to the times.

MOTION OF THE PLANETS AND SATELLITES DUE TO UNIVERSAL GRAVITY

Prop. 1-3: The forces by which the satellites of Jupiter (resp. the primary planets, the moon) are directed away from rectilinear motions and maintained in their respective orbits are directed to the center of Jupiter (resp. to the sun, the center of the earth) and are inversely as the square of their distances from the center.

Prop 4-5. The moon gravitates toward the earth, and by force of gravity is always drawn back from rectilinear motion and kept in its orbit; *mutatis mutandis* for the satellites of Jupiter and the primary planets.

Prop. 6. All bodies gravitate toward each of the planets, and at any given distance from any one planet, the weight of an arbitrary body toward that planet is proportional to the quantity of matter the body contains.

Prop. 7. Gravity exists in all bodies universally, and is proportional to the quantity of matter in each.

Prop. 8. If two globes (with spherically symmetric matter distribution) gravitate toward each other, then the weight of either toward the other will be inversely as the square of the distance between their centers.

Prop. 9. In going inward form the surfaces of the planets, gravity decreases very nearly in the ratio of the distances from the center.

Prop. 13. The planets move in ellipses that have a focus in the center of the sun, and by radii drawn to the center describe areas proportional to the times.

Prop. 15-16. To find the principal diameters, eccentricities and aphelia of the planetary orbits.

Prop. 18-19. The axes of the planets are smaller than the diameters that are drawn perpendicularly to those axes/ problem: to find the ratio of a planet's axis to the diameter proportional to that axis.

Prop. 20. To find and compare with one another the weights of bodies in different regions of our earth.

THE MOON AND THE TIDES

Proposition 24: the ebb and flow of the sea arise from the actions of the sun and the moon.

Proposition 27: From the hourly motion of the moon, to find its distance from earth.

Proposition 35: To find the inclination of the plane of the moon's orbit to the plane of the ecliptic, at a given time.

Proposition 36-37: To find the force of the sun (resp. the moon) to move the sea.

Proposition 38: To find the figure of the body of the moon.

THE COMETS

Proposition 40: Comets move in conics having their foci in the center of the sun, and by radii drawn to the sun describe areas proportional to the times.

Proposition 41: To find the trajectory of a comet moving in a parabola, from three given observations.

Proposition 42: To correct a comet's trajectory that has been found [by the method of proposition 41].

GENERAL SCHOLIUM

[This is primarily a lengthy theological statement; towards the end, N. says:] `This concludes the discussion of God, and to treat of God from phenomena is certainly a part of natural philosophy.'

`Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity…I have not yet been able to deduce from phenomena the reason for these properties of gravity, and I will not pretend to have hypotheses to advance (*Hypotheses non fingo.*)'

BOOK 2: THE MOTION OF BODIES

Book 2 deals with motion in a resistive medium.

SECTIONS 1-4: MOTION UNDER VARIUS HYPOTHESES ON HOW RESISTANCE DEPENDS ON VELOCITY (Props. 1-18)

SECTION 5: DENSITY AND PRESSURE IN FLUIDS, HYDROSTATICS (Props. 19-23)

Prop. 23: If the density of a fluid composed of particles which repel one another is proportional to the pressure, the centrifugal forces of repulsion are inversely proportional to the distances between the centers; and, conversely, particles that are repelled from one another by forces inversely proportional to the distances between the centers constitute an elastic fluid whose density is proportional to the pressure.

SECTION 6: MOTION OF SIMPLE PENDULUMS IN RESISTIVE MEDIA (Props. 24-31)

*SECTION 7: THE MOTION OF FLUIDS AND THE RESISTANCE ENCOUNTERED BY PROJECTILES (*Prop 31-34)

Prop. 34: In a rare medium consisting of particles that are equal and arranged freely at equal distances from one another, let a sphere and a cylinder-described with equal diameters-move with equal velocity along the direction of the axis of the cylinder; then the resistance of the sphere will be half the resistance of the cylinder.

SECTION 8: [WAVE] MOTION PROPAGATED THROUGH FLUIDS (Props. 41-50)

Prop. 48: The velocities of pulses propagated in an elastic fluid are as the square root of the elastic force directly, and the square root of the density inversely, provided the elastic force of the fluid is proportional to its condensation.

SECTION 9: THE CIRCULAR MOTION OF FLUIDS [VORTICES] (Props. 51-53)

Scholium of Prop. 53: `Hence it is clear that the planets are not carried along by corporeal vortices…'