MATH 435- Problems on harmonic functions-due 4/10/2002

1.Solve $\Delta u = 0$ in the spherical shell $0 < a < r < b$ in $\mathbb{R}^3$ with boundary conditions $u = A$ on $r = a$, $u = B$ pn $r = b$, where A and B are constants (look for a solution depending only on r).

2.Solve $\Delta u = 1$ in the annulus $a < r < b$ in $\mathbb{R}^2$, with u vanishing on the boundary (that is, $u = 0$ at $r = a$ and $r = b$).

3.Show that there are no solutions of:

$$\Delta u = f \text{ in } D, \quad \frac{\partial u}{\partial n} = g \text{ on } \partial D,$$

for $D \subset \mathbb{R}^3$ bounded, unless:

$$\int_D f dvol = \int_{\partial D} g dA.$$

(*Hint:*divergence theorem).

4.Suppose that u is a harmonic function in the disk $D = \{r < 2\} \subset \mathbb{R}^2$ and that $u = 3 + 5\sin(7\theta)$ for $r = 2$. Without finding the solution, (i)find the maximum value of u in $\bar{D}$; (ii)find the value of u at the origin.

5.Solve $\Delta u = 0$ in the disk $\{r < R\} \subset \mathbb{R}^2$, with boundary condition

$$u = 2 + 3\cos(2\theta) \text{ on } r = R.$$

6.Solve $\Delta u = 0$ in the *exterior* $\{r > R\}$ of a disk of radius R in $\mathbb{R}^2$, with the boundary condition $u = 1 - 2\sin(5\theta)$ on $r = R$, and the condition at infinity that u be bounded as $r \to \infty$. Without this condition, is the solution unique?