MATHEMATICS 435-PROBLEMS-APRIL 25,2002

1. Solve the wave equation in $\mathbb{R}^3$ with initial data:

$$(i) f = 0, \quad g(x,y,z) = y$$
$$(ii) f = 0, \quad g(x,y,z) = x^2 + y^2 + z^2$$

2.Solve the wave equation with initial data: $f = 0; g = A$ if $r < R, g = 0$ if$r > R$. Find the solution in $\mathbb{R}^2$ and in $\mathbb{R}^3$.

3.Solve the 2d wave equation on the unit disk in $\mathbb{R}^2$, with Dirichlet boundary conditions and initial data $f = 1 - r^2, g \equiv 0$.

4.Solve the 2d heat equation in the annular region $\{1 < r < 2\} \subset \mathbb{R}^2$, with $u = B$ on the boundary and initial condition $f(r), f(1) = f(2) = B$.

5.Find a harmonic function in the unit ball in $\mathbb{R}^3$, with value $g = P_l(\cos\varphi)$ on the boundary (in standard spherical coordinates; P_l is the lth Legendre polynomial.

6. Find the solution to the wave equation in the unit ball in $\mathbb{R}^3$ with Neumann boundary conditions and initial conditions $f = r\cos\varphi$ (in spherical coordinates), $g = 0$. (*Hint:* $P_1^0(x) = x$.)

7. Find the eigenvalues/eigenfunctions of the unit ball in $\mathbb{R}^3$ with Neumann boundary conditions, and their multiplicities.

8.Solve the exterior Dirichlet problem: u is bounded, harmonic outside the unit ball in $\mathbb{R}^3$, with radial derivative equal to $-\cos\varphi$ on the unit sphere.