3 Let $A = \bigcap_{n=1}^{\infty} U_n$ with each U_n an open set in the complete metric space (M, d) (so that A is a G_δ set). Let $U = \prod_{n=1}^{\infty} U_n$, let
$\Delta = \{x \in U | x_1 = x_2 = \dots\}$ (every component of x is the same). Let $g : A \to U$ be defined as $g(x) = (x, x, \dots)$ (throughout the rest of this I'll be using $\overline{x}$ to denote $(x, x, \dots)$). I will be using $D(x, y) = \sum_{n=1}^{\infty} 2^{-n} \frac{d(x_n, y_n)}{1+d(x_n, y_n)}$ to denote the metric in the product space U.
To show: g is an embedding with $g(A) = \Delta$.
Proof: Clearly g is injective (if x and y are different, then their images disagree in every component, in particular they disagree in some component so they're not equal).
Note that $\overline{x} \in \Delta$ iff $x \in U_n \forall n \in \mathbb{N}$, so that $x \in A$, so $g(x) = \overline{x} \in \Delta$ iff $x \in A$, i.e. $g(A) = \Delta$.
Note that sequence convergence in Δ is componentwise convergence.
Let $C \subseteq A$ be closed, let $(\overline{x_n}) \to \overline{x}$ be a sequence in $g(C)$ which converges to a point in Δ. Then (x_n) is a sequence in C and since convergence is componentwise, the sequence converges to $x \in A$. But since C is closed, $x \in C$ and therefore $\overline{x} \in g(C)$, so that g is a closed map.
Let $H \subseteq \Delta$ be a closed set, let $(x_n) \to x$ be a sequence in $g^{-1}(H)$ which converges to a point in A. Then $(\overline{x_n}) \to \overline{x}$ is a sequence in H which converges to a limit, and since H is closed the limit $\overline{x}$ is also in H. Therefore $\overline{x} \in H$ so that $g^{-1}(\overline{x}) = x \in g^{-1}(H)$, making $g^{-1}(H) \subseteq A$ a closed set. Therefore g^{-1} is also a closed map, so that g, g^{-1} are both continuous, making g a homeomorphism onto its image Δ. Then since the countable product space of the complete metric space M is complete and Δ is a closed subset of this product, we have that $\Delta = g(A) \approx A$ is a complete metric space, and by homeomorphism (since g is an embedding) we have that A is completely metrizable.