

9 | Any metrizable space is Hausdorff and first countable.

• Induced topology
& distance function.

$$x \neq y \Rightarrow d(x, y) > 0$$

$$\Rightarrow r = \frac{d(x, y)}{4}$$

$$\Rightarrow B_r(x), B_r(y) \text{ open}$$

$$B_r(x) \cap B_r(y) = \emptyset$$

\Rightarrow Hausdorff.

$$x \in X, \mathcal{U}_x \in \mathcal{T},$$

$$\Rightarrow \exists B_r(x) \subset \mathcal{U}_x$$

$$\Rightarrow \exists r_0 \in \mathbb{Q}, r_0 < r \text{ st.}$$

$$B_{r_0}(x) \subset \mathcal{U}_x$$

$$\Rightarrow \mathcal{B}_x := \{ B_r(x) \mid r \in \mathbb{Q} \}$$

is a local basis.