i Let X be a regular space, let $Y \subseteq X$, and let $y \in Y$. Then in X by regularity, for any open neighborhood $U_y \subseteq X$ of y, there exists another open neighborhood $V_y \subseteq \overline{V_y} \subseteq U_y$ of y. Note then that any open neighborhood of y in Y can be written as $Y \cap U_y$ for some open neighborhood U_y of y in X, and $y \in V_y \cap Y \subseteq \overline{V_y} \cap Y \subseteq U_y \cap Y$, and this gives the regularity condition on Y (since the closure of a set $V \cap Y$ in Y is the same as $\overline{V} \cap Y$).

ii If $X = \prod_{\alpha \in \Lambda} X_\alpha$ is regular, then each subspace must be regular, so since each set in the product can be realized as a subspace by fixing each other component to be constant and allowing the component of that set to vary, this is an onto map, so each factor is homeomorphic to a subspace of the product.
If each X_α is regular, then let $x \in X$, let $U_x \subseteq X$ be an open neighborhood about x. Then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Lambda$ such that $U_x = \prod_{\alpha_1,\alpha_2,\ldots,\alpha_n} U_{x,\alpha_i} \times \prod_{\alpha \in \Lambda \setminus \{\alpha_1,\alpha_2,\ldots,\alpha_n\}} X_\alpha$. Then for each U_{x,α_i} find some open neighborhood about x_{α_i} $V_{x,\alpha_i} \subseteq \overline{V_{x,\alpha_i}} \subseteq U_{x,\alpha_i}$, let $V_x = \prod_{\alpha_1,\alpha_2,\ldots,\alpha_n} V_{x,\alpha_i} \times \prod_{\alpha \in \Lambda \setminus \{\alpha_1,\alpha_2,\ldots,\alpha_n\}} X_\alpha$. Then we have

$$x \in V_x \subseteq \overline{V_x} \subseteq U_x$$

so the product space is regular.