3 Normal Spaces

1. Closed subspaces of a normal space are normal

Proof. Let (X, τ) be a normal space. Since X is normal, X is Hausdorff. If $Y \subset X$ is a closed , then Y is also Hausdorff; since Hausdorffness is hereditary. Since Y is Hausdorff, we can choose two disjoint closed subsets in Y. In particular, choose $\overline{U_x^{(1)}}$ and $\overline{U_y^{(2)}}$, where $U_x^{(1)}$ and $U_y^{(2)}$ are disjoint neighborhoods of x and y respectively for $x, y \in Y$.
Since, X is normal, we can choose open subsets $V^{(1)}, V^{(2)} \subset X$ such that $\overline{U_x^{(1)}} \subset V^{(1)}$ and $\overline{U_x^{(2)}} \subset V^{(2)}$. It follows that $\overline{U_x^{(1)}} \subset V^{(1)} \cap Y$ and $\overline{U_x^{(2)}} \subset V^{(2)} \cap Y$ where $\subset V^{(1)} \cap Y$ and $\subset V^{(2)} \cap Y$ are disjoint open sets in Y. □