Call the metric on M d_1 and the metric on N d_2. Let $a \in \overline{A}$ and let $\varepsilon > 0$. Then $\exists \delta > 0$ such that for all $x \in A, d_1(x, a) < \delta \implies d_2(f(x), F(a)) < \varepsilon$. Then let $y \in \overline{A}$ such that $d_1(y, a) < \delta$. Then $\exists \delta_2 > 0$ such that
$\forall x \in A, d_1(x, y) < \delta_2 \implies d_2(f(x), F(y)) < \varepsilon - d_2(f(x), F(a))$.
Then letting $x \in A, d_1(x, y) < \delta_2$ (such an x exists since $y \in \overline{A}$),
$d_2(F(y), F(a)) \leq d_2(F(y), f(x)) + d_2(f(x), F(y))$
$< \varepsilon - d_2(f(x), F(a)) + d_2(f(x), F(a))$
and therefore
$d_2(F(y), F(a)) < \varepsilon$ is implied by $d_1(y, a) < \delta$ for $x, a \in \overline{A}$, making F a continuous function on $\overline{A}$.
Note, though it wasn't used in the proof, that for $x \in A$ $f(x) = F(x)$ since limits are unique and always exist, so it is reasonable to call F an extension of f as they agree on the domain of f.