7 Note that for any polynomial $p(x)$, $\int\limits_0^1 p(x)f(x)dx = 0$. Then use Stone-Weierstrass to approximate $f(x)$ uniformly by polynomials $p_n(x)$ such that $\forall n \in \mathbb{N} \forall x \in [0,1] |f(x) - p_n(x)| < \frac{1}{n}$. Then $|\int\limits_0^1 f^2(x)dx - \int\limits_0^1 f(x)p_n(x)dx| = |\int\limits_0^1 f^2(x)dx| < \frac{1}{n}$ so that $f^2(x)$ integrates to zero. Then $f^2(x)$ is a nonnegative continuous function which integrates to zero on $[0,1]$, so f^2, and therefore also f, is identically zero on the interval.

7.5 The set of n variable polynomials in the space form an algebra satisfying the hypotheses of the Stone-Weierstrass theorem (separates points, nowhere zero), so we can use them to approximate any continuous function. If $f(0) = 0$, then by the proof of lemma 3 (from 21 October 2020), we can fix 0 in our approximating functions from the algebra (in this case the polynomials) so that every $p_j(0) = 0$ for the approximating functions $p_j(x) \to f(x)$ u.o.c.

7.7 It is enough to show that the lattice $\mathcal{A} \subseteq C(\mathbb{R})$ of continuous piecewise linear functions admits two point interpolation (and is actually a lattice).
Two point interpolation: this is clearly satisfied by continuous piecewise linear functions, just connect the pairs $(p, c_1), (q, c_2)$ via a line.
Lattice: if $f, g \in \mathcal{A}$, then $max\{f, g\} = \frac{1}{2}(f + g + |f - g|) \in \mathcal{A}$, $min\{f, g\} = \frac{1}{2}(f + g + |f + g|) \in \mathcal{A}$, and as long as $\mathcal{A}$ is closed under absolute value, it is closed under max and min and is therefore a lattice. If a linear part falls entirely below the axis, the absolute value negates it and so it remains linear. If a part crosses the axis, then both pieces are linear and they meet where the function hits zero, so the absolute value stays in $\mathcal{A}$, so this is a lattice.
Then by lemma 3 (from class on 21 October), since $\mathcal{A}$ is a lattice admitting two point interpolation, it is dense in $C(\mathbb{R})$.