i Let $\mathcal{U}$ be a collection of disjoint open subsets of $\mathbb{R}^n$ (throughout all three parts I'll be assuming that every set in such a collection is nonempty). Then for each $U \in \mathcal{U}$, U contains at least one point with rational components by density of $\mathbb{Q}^n$ in $\mathbb{R}^n$. For each U, pick a rational point $q_U \in U$. Then the function mapping $U \to q_U$ is an injection (since the open sets are disjoint) from $\mathcal{U} \to \mathbb{Q}^n$, so $\mathcal{U}$ can't be uncountable.

ii Let $f : I \to \mathbb{R}$ be an increasing function with I an interval. Let x be a point of discontinuity. Then $\exists \delta_x > 0$ such that $\forall y > x, f(y) > f(x)+\delta_x$ or $\forall y < x, f(y) < f(x) - \delta_x$. Then find such a δ_x for each point of discontinuity, and note that the intervals $(f(x), f(x) + \frac{\delta_x}{2})$ (if the first holds for x) and $(f(x) - \frac{\delta_x}{2}, f(x))$ (if the second holds for x) are disjoint open intervals indexed by the set of discontinuity, so since any collection of disjoint open intervals is countable, so is the set of discontinuity.

iii Let $U \subseteq \mathbb{R}$ be an open set. Then since $\mathbb{R}$ is locally connected, we have from problem 10 that every connected component of U is an open set. Let $\mathcal{U}$ be the collection of connected components of U. Then from the first part of this problem, $\mathcal{U}$ is countable since it is a collection of disjoint open sets, so U can be expressed in this way.
If $\mathcal{V}$ is another such collection, note that every set in $\mathcal{U}$ must be covered by sets in $\mathcal{V}$, so if $\exists B \in \mathcal{U}, A_1, A_2 \in \mathcal{V}$ such that $B = A_1 \cup A_2$, then these sets disconnect B since they are disjoint, contradicting that B is a connected component. If instead $B \subset A_1 \cup A_2$ and both intersect nontrivially with B, then we have that $(B \cap A_1), (B \cap A_2)$ disconnect B, again a contradiction. If $B \subset A_1$, then we have that A_1 is a connected set in U which properly contains the connected component B, contradicting the definition of connected component. Note that the pair A_1, A_2 could actually be any collection of disjoint open sets (nonempty) and the arguments would still work exactly the same way, I just used pairs to simplify the notation.
Therefore B cannot intersect more that one set from $\mathcal{V}$ and the set it does intersect must be equal to B, and clearly it must intersect at least one set in order for $\mathcal{V}$ to still cover U. Then since every set is nonempty, we have that $\mathcal{U} = \mathcal{V}$, which is the uniqueness we wanted to show.