Let $\mathcal{B} = \{B_1, B_2, \dots\}$ be a countable basis for the second countable space X. Then we seek a countable local basis for each point $x \in X$ to show that X is first countable.
Let $x \in X$. Then let V_x be any open neighborhood about x. Now consider the subcollection of $\mathcal{B}$ we call $\mathcal{C}_x$ defined as the collection of elements of $\mathcal{B}$ which contain x. Then since $\mathcal{B}$ is a basis for the topology, $\exists B_i \in \mathcal{B}$ such that $x \in B_i$ and $B_i \subseteq V_x$, so this set is also in $\mathcal{C}_x$, so the collection $\mathcal{C}_x$ forms a local basis about x, and since it is a subset of the countable set $\mathcal{B}$, it is a countable local basis for x, making the space first countable.
To show X is a separable space, for every nonempty set $B_i \in \mathcal{B}$, find exactly one $x_i \in B_i$ and consider the necessarily countable collection F of these x_i. This set is dense as if y is any point in X with an open neighborhood V_y, then there is some $B_i \in \mathcal{B}$ such that $B_i \subseteq V_y$ and $y \in B_i$. Then there is some element $x_i \in F$ such that $x_i \in B_i \subseteq V_y$, so $x_i \in V_y$, and therefore every open neighborhood of each point y in X contains some element of the countable set F, so F is a countable dense subset of X, and therefore X is separable.