

MATH 561, FALL 2022–PROBLEM SET 2 (due Tuesday 9/20)

**Problem 1.** [Munkres EDT, Problem 1.10 p.11] If  $f : M \rightarrow N$  is a  $C^r$  diffeomorphism, then  $f^{-1}$  is a  $C^r$  diffeomorphism. (Follow the steps indicated in the book to prove the result in euclidean space, then explain why this implies the result for maps of manifolds.)

**Problem 2.** (i) Show that a continuous proper map  $f : M \rightarrow N$  (differentiable manifolds) is a closed map; use this to prove that an injective proper immersion  $f : M \rightarrow N$  is an embedding.

(ii) Show that if  $M$  is compact, any injective proper immersion  $f : M \rightarrow N$  is an embedding.

**Problem 3.** Let  $M$  be a  $C^r$  manifold ( $r \geq 2$ ),  $TM$  its tangent bundle. Show that the differential of the identity map of  $M$  is the identity map of  $TM$ .