

Math 562, Topology

Set 5

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Problem 2: If $m < p$, show that every smooth map $M^m \rightarrow S^p$ is homotopic to a constant.

Solution:

If $m < p$, smooth maps from $M^m \rightarrow S^p$ cannot have regular values in their image because their differential is a linear map from an m dimensional vector space to a p dimensional vector space. Thus smooth maps cannot be surjective by Sard's theorem. But then, if $z \in S^p$ is not in the image, the image of the smooth function is contained in $S^p \setminus z$ which is contractible. Therefore, smooth functions are homotopic to a constant. ■