8 Recall the ε neighborhood theorem (page 69 of G-P), which states that if Y is a compact manifold without boundary (as ∂W is) in $\mathbb{R}^M$, and we let Y^ε be the set of points in $\mathbb{R}^M$ which are within distance ε of Y, then for ε small enough (but still positive), each point $y \in Y^\varepsilon$ has a unique closest point in Y, which is then assigned by the function $\pi(y)$. π is additionally a submersion.

Let then W be embedded in the Euclidean space $\mathbb{R}^M$, define $\pi : \partial W^\varepsilon \to \partial W$ as above. Define now $g = f \circ \pi : \partial W^\varepsilon \to \mathbb{R}^{k+1}$. Let now $\phi : \mathbb{R}^M \to \mathbb{R}$ be a smooth bump function such that $\phi|_{\partial W} \equiv 1$ and $\phi|_{\mathbb{R}^M \setminus K} \equiv 0$ for some $K \subseteq \partial W^\varepsilon \setminus \partial W$ which is also a compact manifold without boundary enclosing some portion of the region enclosed by ∂W.

Define now h as $h(x) = \begin{cases} \phi(x)g(x) & x \in \partial W^\varepsilon \\ 0 & \text{otherwise} \end{cases}.$

Then h extends f to all of $\mathbb{R}^M$, so let $F = h|_W$, and this F is the desired extension.