

MATH 562: TOPOLOGY II, Spring 2020

Outline

- 1- Differentiable manifolds, differentiable maps, diffeomorphisms. Tangent space, tangent bundle, differential of a map. Submanifolds: images of embeddings, preimage of a regular value. Transversality. Partitions of unity--existence (paracompactness) and applications: extension of maps to \mathbb{R}^n , embeddings into \mathbb{R}^n , Riemannian metrics.

Ref: Guillemin-Pollack ch. 1, Munkres 2, ch 1.1, 1.2, Milnor 1, ch 1,2

- 2- Whitney topology. Stability of immersions, submersions, embeddings, diffeomorphisms. Smoothing of maps and manifolds. Generic properties. Sard's theorem. Whitney's immersion and embedding theorems.

Ref: Guillemin-Pollack ch.1, Munkres 2, ch 1.2, 1.3, lecture notes.

- 3- Intersection, degree and application- classification of 1-manifolds, mod 2 intersection theory. Winding numbers, Jordan-Brouwer separation theorem, Borsuk-Ulam theorem. Orientation, oriented intersection number. Vector fields, Poincaré-Hopf theorem. Hopf degree theorem.

Ref: Guillemin-Pollack ch, 2+3, Milnor 1: ch. 4,5,6

- 4- Fundamental group and covering spaces- existence of covering spaces. Covering transformations, universal cover. Properly discontinuous group actions. Seifert-van Kampen theorem and applications. Fundamental group of surfaces and of classical matrix groups. Brouwer's invariance of domain theorem.

Ref: Munkres 1—ch. 11 and sections 62, 74.

- 5- Morse Theory (time permitting). Existence of Morse functions, gradient flow and cell decomposition, use in the classification of surfaces.

Ref: Milnor 2 (ch.1), Hirsch (ch.6), Guillemin-Pollack (ch 1.7)