Topology HW Problem 14

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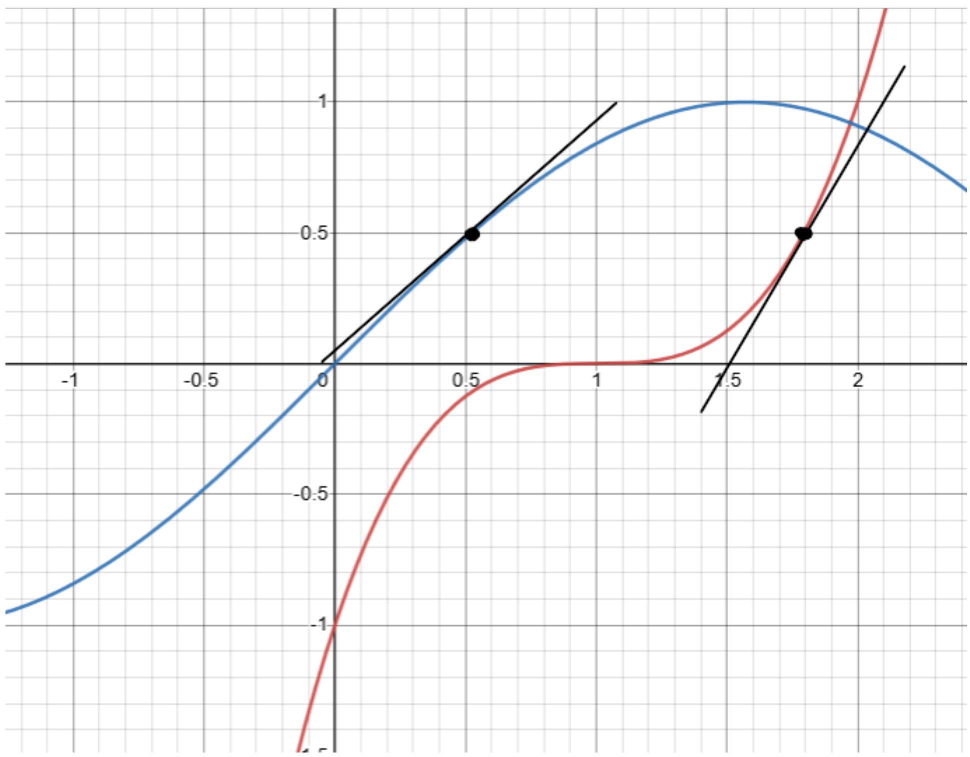
14). We have the definition of two differentiable maps, $f : M \to P$, $g : N \to P$ transversal as

$$df(p)[T_pM] + dg(q)[T_qN] = T_rP$$

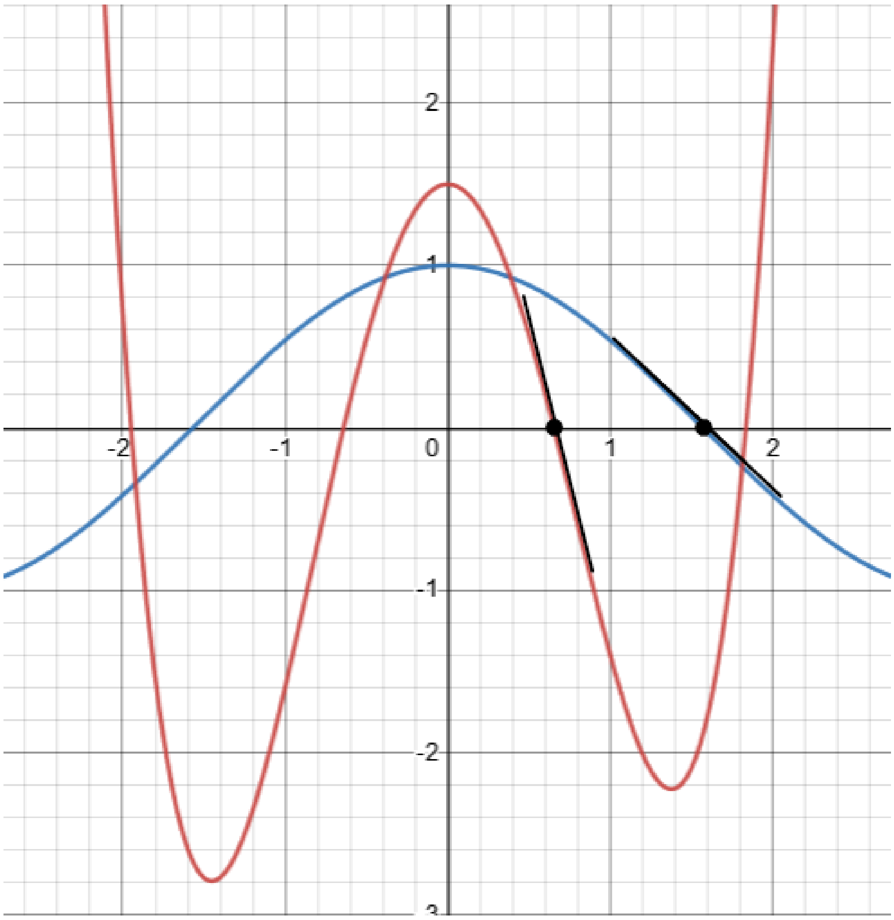
when $f(p) = g(q) = r$.

Now, consider the following example, where $f(x) = \sin(x)$ and $g(x) = x^3$.



We have that $f(\sqrt[3]{0.5}+1) = 0.5$ and that $g(\arcsin 0.5) = 0.5$. For two functions to be transversal to each other, we must have that $f'(p)$ and $g'(q)$ are not *both* equal to zero. As we can see in the picture above, that is the case. Therefore, their tangent spaces span the entire ambient

tangent space. Thus, $f \overline{\pitchfork}_{p,q}\, g$.
Further, we can see that $\Gamma_f \overline{\pitchfork} \Gamma_g$. The two submanifolds intersect at a point, so $codim(\Gamma_f \cap \Gamma_g) = codim(\Gamma_f) + codim(\Gamma_g)$. There would be no way in $\mathbb{R}^2$ to separate the graphs of these two functions and eliminate the intersection.



Now, here we have $f(x)$ is some convoluted function of degree 4, and $g(x) = \cos(x)$. Look at the highlighted points. We have that $f(t_0) = g(t_1) = 0$, and we have that $f'(t_0)$ and $g(t_1)$ are both not zero, and hence the functions are transversal. However, we could shift $f(x)$, the red function, upwards and eliminate the intersections. Hence, the graphs are not transversal and submanifolds. So, we can say that transversality of the graphs does not imply the functions are transversal.