i We have a countable disjoint union of the coordinate chart domains $V_{i,1}, V_{i,2}, \ldots$ with the charts $k_{i,j} : V_{i,j} \to B_{i,j} \subseteq \mathbb{R}^m$, where $B_{i,j} = k_{i,j}(V_{i,j})$ are bounded open sets.
To make the sets disjoint, note that by boundedness we may assume first that each is contained in $B(0,1) \subseteq \mathbb{R}^m$. Now define
$\tilde{k_{i,j}}(p) = k_{i,j}(p) + (2j, 0, 0, \ldots, 0)$, shifting the image along the x_1 axis in $\mathbb{R}^m$ by $2j$. Then we have that $\tilde{k_{i,j}}(V_{i,j}) \subseteq B((2j, 0, \ldots, 0), 1)$, which is disjoint from $B((2\ell, 0, \ldots, 0), 1)$ if $j \neq \ell$.
Now define $h_i : U_i \to \mathbb{R}^m$, where $U_i = \bigcup_{j=1}^{\infty} V_{i,j}$ by $h_i|_{V_{i,j}} = \tilde{k_{i,j}}$ for each $j \in \mathbb{N}$.

ii The three domains I use will be the torus with a ribbon excluded in the horizontal loop and a band excluded in the vertical loop, the ribbon excluded from the first with a part of the excluded loop also excluded, disjoint union a patch on the other side of the loop excluded from the first, and the third domain is the rest of the loop excluded from the first domain. Note that these are all surfaces with no holes and no closed loops, so they are homeomorphic to subsets of Euclidean space.
For notation, I will use $T = S^1 \times S^1$ and parametrize each of the unit circles by a single variable which ranges from $[0, 2\pi)$ (in a few places the value will exceed 2π, in which case the value is reduced by 2π, for example, 3π and π are the same value).
$U_1 = (0, \frac{3}{2}\pi) \times (0, \frac{3}{2}\pi)$
$U_2 = (\pi, \frac{5}{2}\pi) \times (0, \frac{3}{2}\pi) \cup (0, \frac{1}{2}\pi) \times (\pi, \frac{5}{2}\pi)$
$U_3 = (\frac{1}{4}\pi, \frac{17}{8}\pi) \times (\pi, \frac{5}{2}\pi)$