2 Intersection and Boundaries

Problem 1. Show that if F does not hit z, then $W_2(f, z) = 0$

Theorem. If $X = \delta W$ and $f : X \to Y$ may be extended to all of W, then, $\deg_2(f) = 0$

Proof. Since F doesn't hit z; then $z \in \mathbb{R}^n \setminus F(D)$. The unit vector $u : X \to S^{n-1}$ can be defined thus:

$$u(x) = \frac{f(x) - z}{|f(x) - z|}$$

We can also define another unit vector:

$$U(x) = \frac{F(x) - z}{|F(x) - z|}$$

. (We can make this second definition because z is not in the image of F)

It remains to see that U is a smooth extension of u to all of D. Its smoothness comes from the fact that we can write it as a composition of smooth functions

$$U(x) = f \circ g$$

Where $f : x \to \frac{x}{|x|}$ and $g : x \to F(x) - z$. Also note that $g \neq 0$ due to our assumption that $z \in \mathbb{R}^n \setminus F(D)$

Thus, by the theorem above, $W_2(f, z) = \deg_2(u) = 0$ □