

8 If  $f$  were not injective, then there would be some distinct  $x, y \in V$  such that  $f(x) = f(y)$ . Note that since  $\phi : V \rightarrow \phi(V)$  is a homeomorphism, it is a bijection, so there are  $a, b \in \phi(V)$  such that  $\phi^{-1}(a) = x, \phi^{-1}(b) = y$ . Then

$$(a, 0) = (\psi \circ f \circ \phi^{-1})(a) = (\psi \circ f)(x) = (\psi \circ f)(y) = (\psi \circ f \circ \phi^{-1})(b) = (b, 0)$$

but clearly  $a \neq b$ , a contradiction, so  $f$  is injective.

Let  $U \subseteq V$  be an open set. Note that  $\phi(U)$  is open in  $\phi(V)$  (homeomorphism), with  $\Phi(\phi(U))$  open in  $\Phi(\phi(V))$  (the set  $U \times \{0\}$  is open in  $V \times \{0\}$ ). Then  $\Phi(\phi(U)) = \psi(f(U))$  is open in  $\Phi(\phi(V))$  if and only if  $f(U)$  is open in  $f(V)$  under the induced topology on  $f(V)$  from  $Z$  since  $\psi$  is a diffeomorphism.