M663 ALGEBRAIC TOPOLOGY I: COURSE OUTLINE.

PART 0: GEOMETRIC NOTIONS [Hatcher, ch. 0]

CW complexes/ CW pairs (topology: Appendix)/ operations on spaces/ criteria for homotopy equivalence/ homotopy extension property.

PART 1: HOMOLOGY [Hatcher, ch. 2]

2.1 Singular, simplicial and cellular homology.

simplicial homology of Δ-complexes, H_n^{Δ}/ simplicial complexes
singular homology, $H_n(X)$: $H_0(X)$, reduced homology $\tilde{H}_n(X)$, homotopy invariance
(2A) first homology and fundamental group
long exact sequence, relative homology (appln: invariance of dimension)
excision theorem, barycentric subdivision
equivalence of simplicial and singular homology: $H_n^{\Delta}(X) \sim H_n(X)$, for a Δ-complex X.
(2C) simplicial approximations: Lefschetz maps, CW complexes
Existence of triangulations of manifolds [Munkres, thm 10.6]/ manifolds homotopically equivalent to CW complex [Hirsch ch.6, via Morse theory]

(2B), 2.2: Topological applications.

(2B) Jordan-Brouwer separation theorem, invariance of domain, Schönflies theorem [see also Bredon, IV.19], Borsuk-Ulam theorem
(2.2) cellular homology of CW complexes: isomorphism $H_n^{CW}(X) \sim H_n(X)$; examples
Euler characteristic (of finite CW complexes)
Mayer-Vietoris sequence

PART 2: DE RHAM COHOMOLOGY [Bott-Tu, ch. 1, no. 1–5].

1. The de Rham complex and compact support
2. Mayer-Vietoris sequence
3. Orientation and integration of differential forms
4. Poincaré lemmas
5. Finite good covers and finite dimensionality/ Poincaré duality for de Rham cohomology (orientable manifolds) /Künneth formula for de Rham cohomology/ Poincaré dual of a closed submanifold

PART 3: COHOMOLOGY AND DUALITY [Hatcher, ch.3]

(3.1) Cohomology of spaces
(3.2) Cup product in cohomology/ cohomology ring/ Künneth formula
(3.3) Poincaré duality

[Bredon V.9] de Rham's theorem
[Bredon V.11] Hopf's theorem on maps to S^n