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Math 231:Introduction to Ordinary Differential Equations
Mini-Project: Insurgencies

Imperial powers frequently find themselves in a situation where they have defeated the military force of a nation, and now want to occupy the land of that nation. However, the people of that nation are hostile and resent the presence of the occupiers. The natives thus form resistance movements to make life unpleasant and dangerous for the occupiers. The native insurgent forces have no chance of forcing the occupiers out of the land; instead their goal is simply to cause problems for the occupiers in the hope that eventually public opinion or a loss of will causes the occupiers to withdraw.

We will look at two models of the numbers $\omega(t)$ of occupiers and $i(t)$ of insurgents. The first is

$$\frac{di}{dt} = (a + d\omega - ei)i - Fi\omega \ , \tag{1}$$

$$\frac{d\omega}{dt} = r(C - \omega)i \ . \tag{2}$$

The number of insurgents tends to increase due to recruitment by existing insurgents (measured by a) and due to the presenence of the occupiers. The parameter d represents a measure of how much the resentment and antagonism of the presence of an occupier excites among the native population contributes to the growth of the insurgent force per unit time. On the other hand, when insurgents interact with the occupiers, they may be killed, captured, or otherwise neutralized; or again, they may be bought off or convinced in some other way not to continue their activities. The term Foi describes the interaction of the insurgents with the occupiers and it's effect toward decreasing the insurgent forces. We also add a term that models the idea that there cannot be an unlimited number of insurgents, so we have the $-ei^2$ term that gives the insurgents logistic growth rather than exponential growth in the absense of the occupiers.

The second equation indicates that the number of occupiers tends to rise as long as there are insurgents to deal with, but there is a limit to the size of the occupying forces. The maximum size the occupying forces are willing to take on is described by $C > 0$, , and the coefficient r

gives a kind of measure of the level of responsiveness of the occupying forces in changing their size to deal with an insurgent threat.

The struggle these equations describe is sometimes called "asymmetric warfare", and that asymmetry is reflected in the equations. It is assumed that the insurgents cannot really do much damage to the occupiers, so we neglect any reduction of occupying forces due to actual interaction with the insurgents.

The second model is

$$\frac{di}{dt} = (a + d\omega - ei)i - Fi\omega \; , \tag{3}$$

$$\frac{d\omega}{dt} = r(C - \omega)i - Ki\omega \; . \tag{4}$$

The parameters and terms we had before still have the same meaning, but now we've added a term that models the damage the insurgents can do to the occupiers, which has the same structure as the term that modeled the damage the occupiers could do to the insurgents.

I have set up this project to be able to accomodate four members. Each member should have one of numbers 1, 2, 3, or 4 that they are responsible for, and 5 should be done together as a group.

1. Do a full phase plane analysis for the first model equations. State the equillibrium points of these equations, and interpret them in terms of the numbers of insurgents and occupiers. What are the possible long term outcomes according to the phase plane? In this analysis, let $F = 0.3$, $a = 8$,$C = 500$, $d = 0.2$ and $r = 0.1$, and $e = 0.01$. Use MATLAB to plot 3 different trajectories for 3 different inital conditions (use initial conditions from different regions of the phase plane to obtain qualitatively different results if possible). What do these results say about the outcomes of these particular initial occupation strategies you've chosen?

   Repeat the work for $F = 0.15$ instead. How do our expectations change? If there are differences in the long term outcomes for this case as compared to the case for $F = 0.3$ above, explain why those differences arise in terms of the change in coefficient. Make sure to include what this means in terms of the situation on the ground.

2. Now we'll investigate the effect that d has on the outcome. First let $d = 0.1$ with all other coefficients set as in number 1. What does this change in coefficient represent? Repeat the instructions for the first part of number 1 for this situation. What differences are there,

if any in the dynamics of the system as compared with the results for the first part of number 1? Explain any differences you find in terms of the meaning of the changes in the situation on the ground. If there are no differences, explain why.

Next, let $d = 0.4$ and repeat the process. What does this represent? What differences are there, if any, in the dynamics of the system as compared with the results for when $d = 0.1$ and $d = 0.2$? Explain any differences you find in terms of the meaning of the changes in the situation on the ground. If there are no differences, explain why.

3. For this part we'll use the second model system of equations where the insurgents are able to cause loses in the occupying forces. Using the same coefficients in part 1 of problem 1, with $K = .1$, and repeat the steps for part 1 of problem 1. What does this choice of values for K represent?

   What differences are there, if any in the dynamics of the system as compared with the results for when there is no K term? Explain any differences you find in terms of the meaning of the changes in the situation on the ground. If there are no differences, explain why.

   Now let $K = 0.3$ so that the insurgents are just as effective as the occupiers at suppressing the other side. Repeat the steps above, leaving all other parameters the same. Again, explain any differences you find in terms of the meaning of the change in the situation on the ground. If there are no differences, explain why.

4. One of the strategies used in the war in Afghanistan had been "winning the hearts and minds of the people". This would correspond to making our coefficient d negative. Let's investigate how would $d < 0$ would effect the dynamics of the systems.

   Again, use the same parameters given in the second part of problem 1 (so $F = 0.15$), and instead let $d = -0.1$. Do the full phase plane, find equillibria, and list all possible long term outcomes. Use MATLAB to plot at least 3 different trajectories for three different initial conditions, chosen from different regions of your phase plane, if possible. What would we expect to see over time if we could actually create conditions represented by $d < 0$? Is there much difference between these results and the results for the second part of problem 1?

   Let's compare these results with leaving $d = 0.2$ and instead increasing our occupier capacity C. Let $C = 1000$, so that we've double our maximum number of occupiers, but the community is still antagonized by their presence. Redo the steps above and compare. Which strategy is more effective in the long run?

5. It turns out that one of the critical pieces of information that determines whether or not the insurgency can be controlled is whether or not $F - d$ is positive or negative. Is this claim consistent with your results? What do you see when $F - d > 0$ What about when $F - d < 0$?

   Summarize your findings, and make recommendations based on what you've learned for how to best manage an occupying force to guarantee that an insurgency is ended, if it's possible, and discuss when it may or may not be possible.