1. True or False? If true, give a complete explanation for why it is true. If false, explain why or give a counterexample.

(a) [8 points] If $\vec{a} \perp \vec{b}$, then $\vec{a} \cdot \vec{b} = 0$.

If $\vec{a} \perp \vec{b}$, then the angle between $\vec{a}$ and $\vec{b}$ is $90°$, or $\pi/2$.

Since $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$, and $\cos(90°) = 0$,

then $\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(90°) = 0$,

So it's true.

(b) [8 points] If $\vec{a} \perp \vec{b}$, then $\vec{a} \times \vec{b} = \vec{0}$.

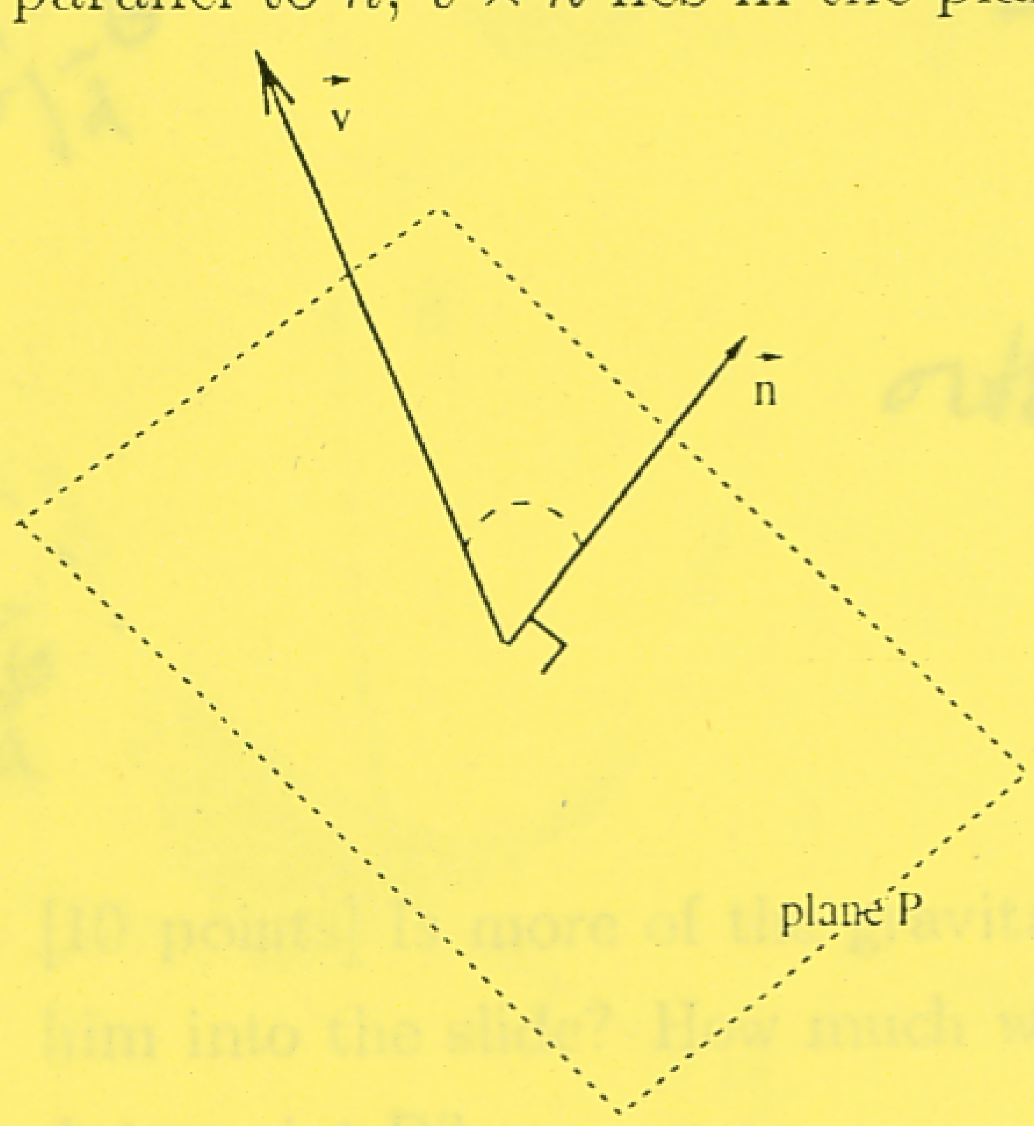
$\vec{a} \perp \vec{b} \Rightarrow$ angle θ between $\vec{a}$ and $\vec{b}$ is $90°$,

so $\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}|\sin\theta)\vec{n} = (|\vec{a}||\vec{b}|\sin(90°))\vec{n}$

$= |\vec{a}||\vec{b}|\vec{n}$

which is not necessarily $= \vec{0}$. So, it's false.

(c) [7 points] Given a plane P with normal vector $\vec{n}$, and any other vector $\vec{v}$ which is not parallel to $\vec{n}$, $\vec{v} \times \vec{n}$ lies in the plane P.



* $\vec{n} \times \vec{v}$ is $\perp$ to $\vec{n}$ and $\perp$ to $\vec{v}$.

Since $\vec{n} \times \vec{v}$ is $\perp$ to $\vec{n}$,

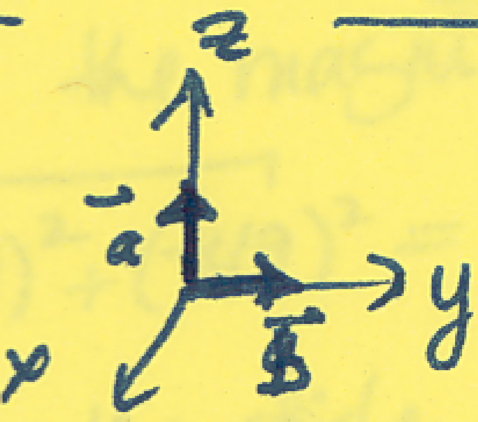
it must lie in the plane P,

because $\vec{n}$ is $\perp$ P and

P contains all vectors $\perp$ to $\vec{n}$.

So it's true.

> part (b) by counterexample:



Let $\vec{a} = \langle 0,0,1 \rangle$ and $\vec{b} = \langle 0,1,0 \rangle$

then $\vec{a} \perp \vec{b}$ since $\vec{a}$ lies on the z-axis and $\vec{b}$ lies on the y-axis.

but $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \langle -1, 0, 0 \rangle \neq \vec{0}$. So the statement is false.