4. Given two lines in vector form

$$\vec{r}_1(t) = <1, 0, 1> + t <1, -2, 2>$$

$$\vec{r}_2(s) = <4, -2, 0> + s <2, 0, -3>$$

(a) [8 points] Find the point of intersection of these two lines, if it exists.

$\vec{r}_1: \; x = 1+t, \quad y = -2t, \quad z = 1+2t$

$\vec{r}_2: \; x = 4+2s, \quad y = -2, \quad z = -3s$

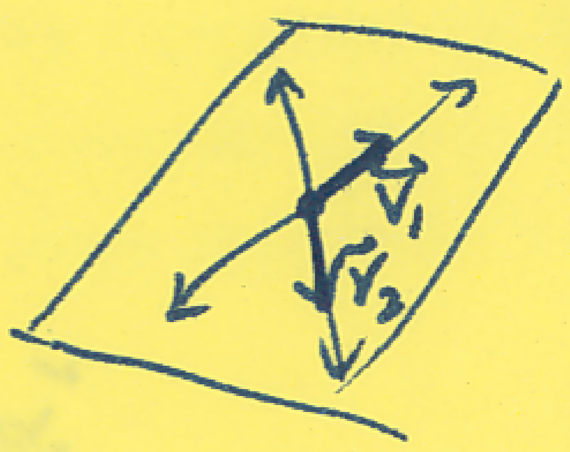
$\Rightarrow \; 1+t = 4+2s$
$-2t = -2 \Rightarrow t = 1$
$1+2t = -3s$

if $t=1$, then
$z = 4+2s$
$2s = -2 \Rightarrow s = -1$

checking last terms:
$1+2(1) \overset{?}{=} -3(-1)$ ✓

$\vec{r}_1(1) = \langle 1,0,1\rangle + 1\langle 1,-2,2\rangle = \langle 2,-2,3\rangle \longrightarrow$ so intersection occurs for $t=1$ and $s=-1$ at point $(2,-2,3)$

(b) [10 points] Find the equation of the plane containing these two lines.



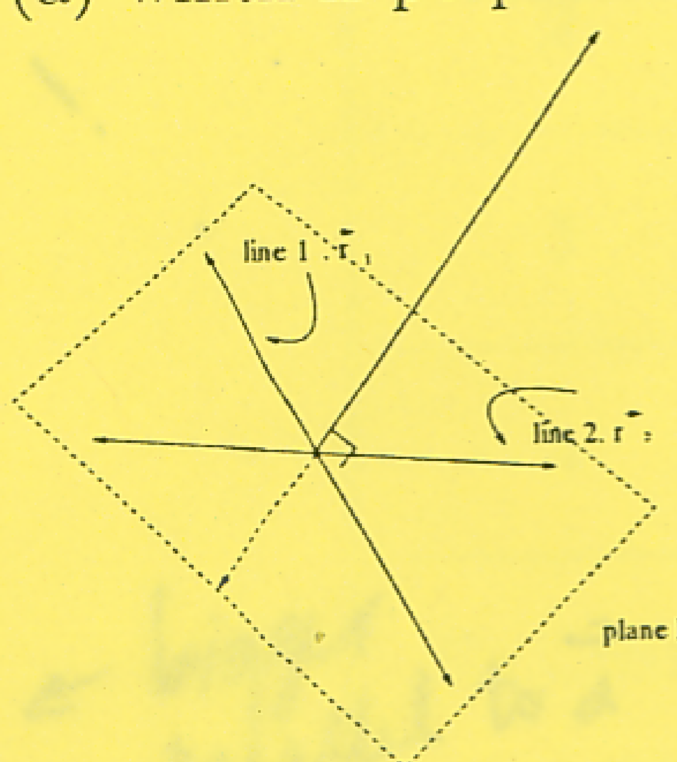
Since $\vec{r}_1$ and $\vec{r}_2$ lie <u>in</u> the plane, if we cross their direction vectors $\vec{v}_1 = \langle 1,2,-2\rangle$ and $\vec{v}_2 = \langle 2,0,-3\rangle$, we get a normal vector to the plane:

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & +2 \\ 2 & 0 & -3 \end{vmatrix} = \langle +6, -(-3-4), 0+4\rangle = \langle 6,7,4\rangle$$

the point $(2,-2,3)$ is on the plane, so the equation is

$$\langle 6,7,4\rangle \cdot \langle x-2, y+2, z-3\rangle = 0$$

(c) [8 points] Find the vector equation of the line through the point you found in part (a) which is perpindicular to the plane you found in part (b).



We know from (b) that $\langle 6,7,4\rangle$ is $\perp$ to the plane, so we can use it as the direction vector for our line:

$$\vec{r}(t) = \langle 2,-2,3\rangle + t\langle 6,7,4\rangle$$

[If you were unable to do (a), pretend the point is $(0, 1, 1)$ in parts (b) and (c). If you were unable to do (b), pretend the plane is $2x - y + z = 3$ in part (c).]