If we want to compute $\int \frac{1}{x}\,dx$,

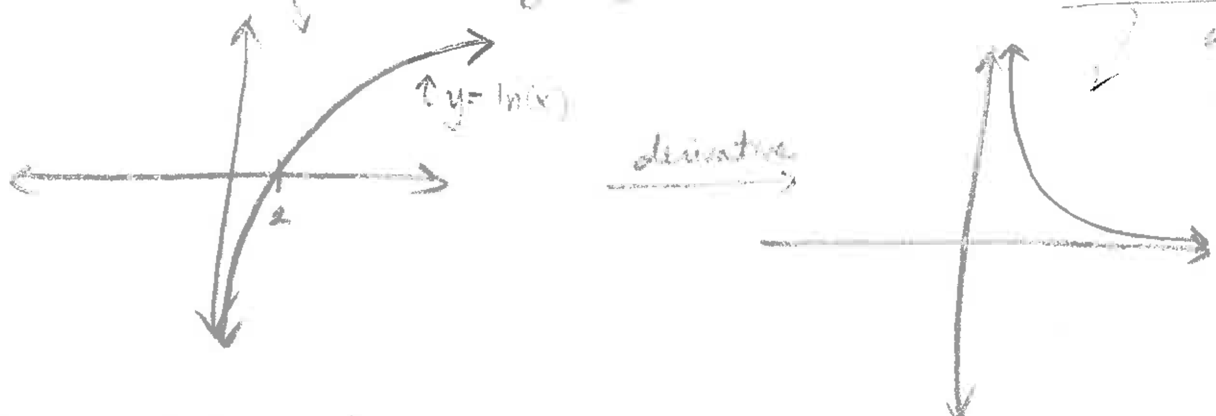
the first thing that comes to mind is that

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}.$$

<u>BUT</u>, we need to be careful here, because <u>really</u> what we should say is that

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x} \quad \underline{\text{for } x>0}.$$

Since $\underline{\ln(x)}$ is only defined for $x>0$, <u>it's derivative</u> is also only defined for $x>0$.

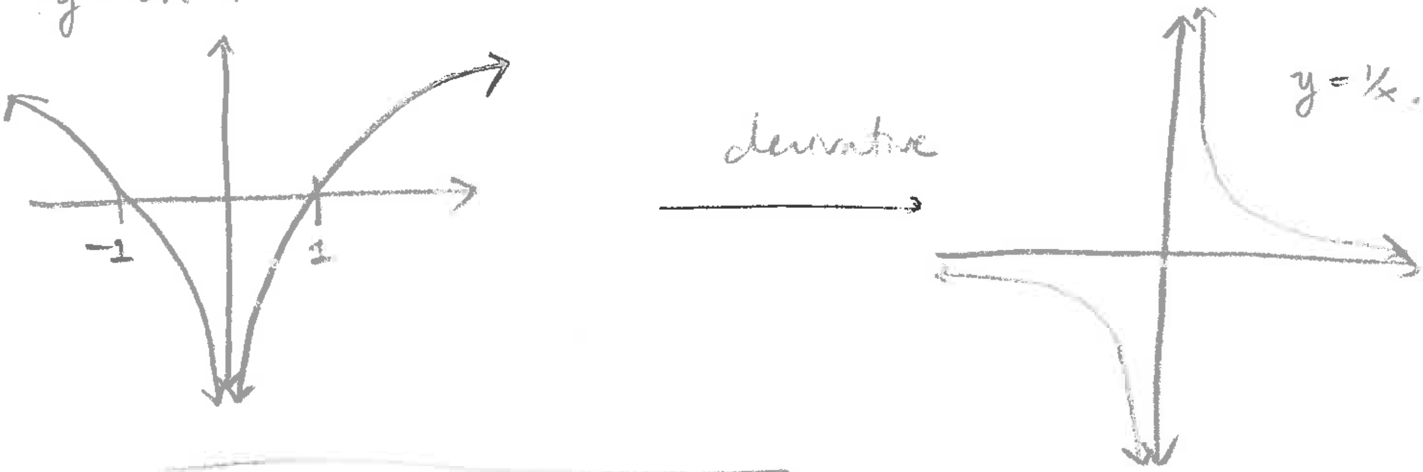


We want to ask though, <u>what function</u> $F(x)$ has as it's derivative $\frac{1}{x}$ (for <u>all</u> $\underline{x \neq 0}$!)?

The answer must clearly be a function defined for all $\underline{x \neq 0}$, rather than just for $x>0$.

The function $\ln|x|$ turns out to be what we need!

$y = \ln|x|$:



So $\boxed{\int \frac{1}{x}\,dx = \ln|x| + C.}$

Finally, it's important to be careful with indefinite integrals of any functions that are not on our list of basic functions.

Common errors:

① $\int 4\,\frac{\sin(x)}{\cos(x)}\,dx \neq 4\,\frac{\int \sin(x)\,dx}{\int \cos(x)\,dx}$

You cannot take the antideriv. of a quotient by finding the quotient of the antiderivatives!!

$\int 4\,\frac{\sin(x)}{\cos(x)}\,dx = -4\ln|\cos(x)| + C$ ← actual antideriv.

$4\,\frac{\int \sin(x)\,dx}{\int \cos(x)\,dx} = 4\left(\frac{\cos(x)+c_1}{\sin(x)+c_2}\right)$ ← NOT the antideriv.

(2)

$$\int f(x)g(x)\,dx \neq \left(\int f(x)\,dx\right)\left(\int g(x)\,dx\right)!$$

Similarly the antideriv. of a product is <u>not</u> equal to the product of the antiderivs.

<u>example:</u>

$$\frac{1}{4}x^4+C \overset{\checkmark}{=} \int x^3\,dx \overset{\checkmark}{=} \int x\cdot x^2\,dx \overset{\times}{\neq} \left(\int x\,dx\right)\left(\int x^2\,dx\right)$$

correct antideriv. of x^3.

$$\left(\int x\,dx\right)\left(\int x^2\,dx\right) = \left(\frac{1}{2}x^2+C_1\right)\left(\frac{1}{3}x^3+C_2\right)$$

<u>not</u> the correct antideriv of x^3!

Just as we did with derivatives, we will need to learn more complex techniques for integrating functions that are not from our basic function list.