Solutions.

Work-It-Out Day 10: Sections 8.1,8.3, 8.4
Math 142 - Spring 2016

1. Calculate the arclength of $f(x) = (x+2)^{3/2}$ for x in $[1, 3]$.

$$A = \int_1^3 \sqrt{1+\left(\frac{3}{2}(x+2)^{1/2}\right)^2}\,dx = \int_1^3 \sqrt{1+\frac{9}{4}x+\frac{9}{2}}\,dx$$

$$= \int_1^3 \sqrt{\frac{11}{2}+\frac{9}{4}x}\,dx = \int_{\frac{31}{4}}^{\frac{49}{4}} \frac{4}{9}\sqrt{u}\,du = \frac{8}{27}u^{3/2}\Big|_{\frac{31}{4}}^{\frac{49}{4}} = \frac{8}{27}\left[\left(\frac{49}{4}\right)^{3/2} - \left(\frac{31}{4}\right)^{3/2}\right]$$

use: $\begin{cases} u = \frac{11}{2} + \frac{9}{4}x \\ du = \frac{9}{4}dx \end{cases}$

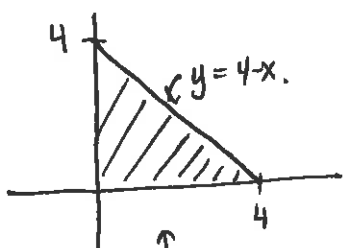
2. Approximate the arclength of $f(x) = \frac{1}{x}$ using S_6 over $[1, 2]$. $\Delta x = \frac{2-1}{6} = \frac{1}{6}$

$1 \quad \frac{7}{6} \quad \frac{4}{3} \quad \frac{3}{2} \quad \frac{5}{3} \quad \frac{11}{6} \quad 2$

$$A = \int_1^2 \sqrt{1+\frac{1}{x^4}}\,dx \approx \frac{1}{18}\left[f(1)+4f\left(\tfrac{7}{6}\right)+2f\left(\tfrac{4}{3}\right)+4f\left(\tfrac{3}{2}\right)+2f\left(\tfrac{5}{3}\right) + 4f\left(\tfrac{11}{6}\right)+f(2)\right]$$

$$\approx \frac{1}{18}\left[\sqrt{2} + 4\sqrt{1+\frac{6^4}{7^4}} + 2\sqrt{1+\frac{3^4}{4^4}} + 4\sqrt{1+\frac{2^4}{3^4}} + 2\sqrt{1+\frac{3^4}{5^4}} + 4\sqrt{1+\frac{6^4}{11^4}} + \sqrt{\frac{17}{16}}\right]$$

3. Find the centroid of the triangle in the first quadrant bounded by the x and y axes and the line $y = 4 - x$.



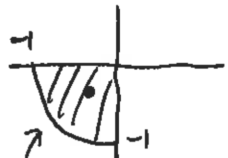
area $= \frac{1}{2}\cdot 4\cdot 4 = 8$.

$$\bar{x} = \frac{\int_0^4 x(4-x)\,dx}{8} = \frac{2x^2 - \frac{1}{3}x^3\Big|_0^4}{8} = \frac{32-\frac{64}{3}}{8} = 4 - \frac{8}{3} = \frac{4}{3}$$

$$\bar{y} = \frac{\int_0^4 y(4-y-0)\,dy}{8} = \frac{2y^2 - \frac{1}{3}y^3\Big|_0^4}{8} = \frac{4}{3}$$

$$\boxed{\text{centroid} = \left(\frac{4}{3}, \frac{4}{3}\right)}$$

4. Find the centroid of the quarter of the unit circle lying in the third quadrant.



$y = -\sqrt{1-x^2}$

has signed area $= -\frac{\pi(1)^2}{4}$.

$u = 1 - x^2$, $du = -2x\,dx$

$$\bar{x} = \frac{\int_{-1}^{0} -x\sqrt{1-x^2}\,dx}{-\frac{\pi}{4}} = \frac{\int_0^1 \frac{1}{2}\sqrt{u}\,du}{-\frac{\pi}{4}} = \frac{\frac{1}{3}\left(u^{3/2}\right)\Big|_0^1}{-\frac{\pi}{4}} = \underline{\underline{\frac{-4}{3\pi}}}$$

$-y\left(0 - \sqrt{1-y^2}\right)$

$$\bar{y} = \frac{\frac{1}{2}\int_{-1}^{0}\left(\sqrt{1-x^2}\right)^2}{-\pi/4} = \frac{\frac{1}{2}\int_{-1}^{0} 1-x^2}{-\pi/4} = \frac{\frac{1}{2}\left(x - \frac{1}{3}x^3\right)\Big|_{-1}^{0}}{-\pi/4}$$

$$= \frac{\frac{1}{2}\left(\frac{2}{3}\right)}{-\pi/4} = -\underline{\underline{\frac{4}{3\pi}}}$$

$$\boxed{\text{centroid} = \frac{-4}{3\pi}, \frac{-4}{3\pi}}$$

5. Find T_2 for $y = e^{\cos(x)}$ at $x = \pi$. Use a calculator to compute the error at $x = 3$.

$$T_2(x) = f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)}{2}(x-\pi)^2$$

$f(\pi) = e^{-1}$

$f'(x) = -\sin(x)e^{\cos(x)} \Rightarrow f'(\pi) = 0$

$f''(x) = \sin^2(x)e^{\cos(x)} - \cos(x)e^{\cos(x)}$

$\Rightarrow f''(\pi) = e^{-1}$

$\Rightarrow \quad T_2(x) = \frac{1}{e} + \frac{1}{e}(x-\pi)^2$

So $\left|f(3) - T_2(3)\right| = \left|e^{\cos(3)} - \left(\frac{1}{e} + \frac{1}{e}(3-\pi)^2\right)\right|$

$= \underline{0.0037}$.

error in T_2 at $x = 3$.

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6. Estimate the maximum error for x in $[0, 0.5]$ for T_2 of $f(x) = \ln(x+3)$, if T_2 is centered at zero.

$$\text{error}(T_2) \le \frac{K(x-0)^3}{3!} \le \frac{\frac{2}{27}(0.5)^3}{3!} = \frac{1}{(27)(24)} = \frac{1}{648}$$

because:

$K \ge |f^{(3)}(u)|$

for all u between x and 0.

Since $x \in [0, 0.5]$, u is in $[0, 0.5]$ too.

$f'(x) = \frac{1}{x+3}$, $f''(x) = \frac{-1}{(x+3)^2}$, $f'''(x) = \frac{2}{(x+3)^3}$

So $\left|f'''(x)\right| = \left|\frac{2}{(x+3)^3}\right| \le \frac{2}{27}$

for x in $[0, 0.5]$