Solutions

Work-It-Out Day 11: Sections 10.1 and 10.2
Math 142 - Spring 2016

1. Determine the limit of the sequence $\{a_n\}$ if it converges, or show that it diverges:

$$a_n = \left(\frac{3}{8}\right)^{-n}$$

diverges to ∞

$$\lim_{n\to\infty} \left(\frac{3}{8}\right)^{-n} = \lim_{n\to\infty} \left(\frac{8}{3}\right)^{n} = +\infty$$

↑ geometric with $r > 1$.

2. Determine the limit of the sequence $\{a_n\}$ if it converges, or show that it diverges, if

$$a_n = \frac{n}{2}\ln\left(\frac{1}{n}\right)$$

diverges

$$\lim_{n\to\infty} \frac{n}{2}\cdot \ln\left(\frac{1}{n}\right) = -\infty$$

($\frac{n}{2} \to +\infty$, $\ln(\frac{1}{n}) \to -\infty$)

3. Determine the limit of the sequence $\{a_n\}$ if it convergesor show that it diverges:

$$a_n = \frac{5^n - 3}{8^n}$$

converges to zero

$$\lim_{n\to\infty} \frac{5^n - 3}{8^n} = \lim_{n\to\infty} \left(\frac{5}{8}\right)^n - 3\left(\frac{1}{8}\right)^n$$

$$= \lim_{n\to\infty} \left(\frac{5}{8}\right)^n - \lim_{n\to\infty} 3\left(\frac{1}{8}\right)^n = 0 - 0 = 0$$

↑ geom. w/ $c=1$, $r=\frac{5}{8}<1$

↑ geometric w/ $c=3$, $r=\frac{1}{8}<1$

4. Compute the following partial sums for the series:

$$\sum_{i=1}^{\infty} \frac{(-1)^i i^2}{i^3+1}.$$

$S_1 = -\frac{1}{2}$

$S_2 = -\frac{1}{2} + \frac{4}{9} = -\frac{1}{18}$

$S_3 = -\frac{1}{2} + \frac{4}{9} - \frac{9}{28} = -\frac{1}{18} - \frac{9}{28} = \frac{-14}{2^2 \cdot 7 \cdot 9} - \frac{81}{2^2 \cdot 7 \cdot 9} = \frac{-95}{28 \cdot 9}$

$= \frac{-95}{252}$

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{5^n - 3}{8^n}$ converges or diverges. If it converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{5^n - 3}{8^n} = \sum_{n=1}^{\infty} \left(\frac{5}{8}\right)^n - 3\left(\frac{1}{8}\right)^n = \sum_{n=1}^{\infty}\left(\frac{5}{8}\right)^n - 3\sum_{n=1}^{\infty}\left(\frac{1}{8}\right)^n$$

↑ geometric series w/ $c=1$, $r = 5/8 < 1$.

↑ geometric series w/ $c=3$, $r = \frac{1}{8} < 1$.

both converge.

$$= \frac{\left(\frac{5}{8}\right)}{1-\frac{5}{8}} - \frac{3\left(\frac{1}{8}\right)}{1-\frac{1}{8}}$$

$$= \frac{5}{3} - \frac{3}{7}$$

So the series converges to $\boxed{\frac{5}{3} - \frac{3}{7}}$

6. Find the value of the sum: $3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \cdots$

$$3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \cdots = \sum_{n=0}^{\infty} \frac{3(-1)^n 2^n}{5^n} = 3\sum_{n=0}^{\infty}\left(\frac{-2}{5}\right)^n$$ ← geometric series w/ $c=3$, $r = -2/5$.

Since $|r| = |-2/5| < 1 \Rightarrow$ it converges to $\frac{3}{1-(-2/5)} = \frac{3}{7/5} = \boxed{\frac{15}{7}}$