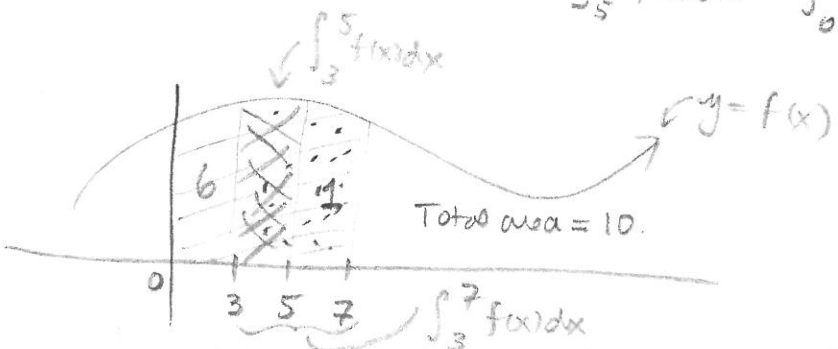
Solutions

Work-It-Out Day 2 Supplemental Problems

1. Suppose that $\int_0^3 f(x)\,dx = 6$, $\int_0^7 f(x)\,dx = 10$, and $\int_5^7 f(x)\,dx = 1$. Compute

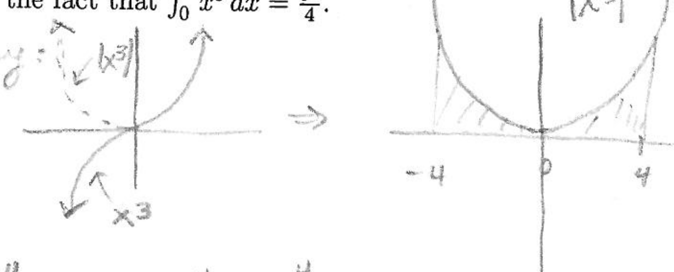
   (a) $\int_3^7 f(x)\,dx = \int_0^7 f(x)dx - \int_0^3 f(x)dx = 10 - 6 = 4$

   (b) $\int_3^5 f(x)\,dx = \int_0^7 f(x)dx - \int_5^7 f(x)dx - \int_0^3 f(x)dx = 10-6-1=3$



2. Compute $\int_{-4}^1 |x^3|\,dx$, using the fact that $\int_0^b x^3\,dx = \frac{b^4}{4}$.

can use symmetry:



$\Rightarrow$ area from -4 to 0 under $|x^3|$ equals area from 0 to 4 under x^3.

$$\int_{-4}^{1} |x^3|\,dx = \int_0^1 x^3 dx + \int_0^4 x^3 dx = \frac{1^4}{4} + \frac{4^4}{4}$$

3. Find the indefinite integral $\int \frac{1}{\sqrt[5]{t^4}}\,dt$.

$$\int \frac{1}{\sqrt[5]{t^4}}\,dt = \int t^{-4/5}\,dt = \frac{t^{1/5}}{1/5} + C = 5t^{1/5} + C$$

typo!

4. Find the general antiderivative of $f(x) = \sqrt[3]{x}(2+\sqrt[4]{x})^2 \cancel{dx}$

$$f(x) = x^{1/3}(2+x^{1/4})^2 = x^{1/3}\left[4+4x^{1/4}+x^{1/4}\cdot x^{1/4}\right]$$

$$= x^{1/3}\left[4+4x^{1/4}+x^{1/4+1/4}\right]$$

$$= x^{1/3}\left[4+4x^{1/4}+x^{1/2}\right] = 4x^{1/3}+4x^{1/4+1/3}+x^{1/2+1/3}$$

$$= 4x^{1/3}+4x^{7/12}+x^{5/6}$$

$$\Rightarrow \int f(x)\,dx = C + \frac{4x^{4/3}}{(4/3)} + \frac{4x^{19/12}}{(19/12)} + \frac{x^{11/6}}{(11/6)} = \boxed{3x^{4/3}+\frac{48}{19}x^{\frac{19}{12}}+\frac{6}{11}x^{\frac{11}{6}} + C}$$

5. If $f''(t) = t^3 + 2t$ and $f'(0) = 3$ and $f(0) = 4$, what functions must $f'(t)$ and $f(t)$ be?

$$f''(t) = t^3+2t$$

$$\Rightarrow f'(t) = \frac{1}{4}t^4+t^2+C$$

so $f'(0) = C = 3.$ $\Rightarrow \boxed{f'(t) = \frac{1}{4}t^4+t^2+3}$

$$\Rightarrow f(t) = \frac{1}{20}t^5+\frac{1}{3}t^3+3t+C$$

so $f(0) = C = 4$ $\Rightarrow \boxed{f(t) = \frac{1}{20}t^5+\frac{1}{3}t^3+3t+4}$