Supplemental Problems - WIOD over 7.1 and 7.5

Name: ____

1.

$$\int \frac{x+1}{x^2+2x+3}\,dx$$

$$\begin{aligned}\int \frac{x+1}{x^2+2x+3}\,dx &= \int \frac{1/2du}{u}\\ &= \frac{1}{2}\ln|u|+C\\ &= \frac{1}{2}\ln(x^2+2x+3)+C\end{aligned}$$

2.

$$\int \frac{x}{x^2+2x+3}\,dx$$

$$\begin{aligned}\int \frac{(x+1)-1}{(x+1)^2+2}\,dx &= \int \frac{1}{2}\frac{du}{u} - \int \frac{1}{w^2+2}\,dw\\ &= \frac{1}{2}\ln|u| - \frac{\sqrt{2}}{2}\int \frac{1}{z^2+1}\,dz\\ &= \frac{1}{2}\ln|x^2+2x+3| - \frac{1}{\sqrt{2}}\arctan(z)+C\\ &= \frac{1}{2}\ln|x^2+2x+3| - \frac{1}{\sqrt{2}}\arctan(\frac{w}{\sqrt{2}})+C\\ &== \frac{1}{2}\ln|x^2+2x+3| - \frac{1}{\sqrt{2}}\arctan(\frac{x+1}{\sqrt{2}})+C\end{aligned}$$

where we let $u = x^2+2x+3$, so that $du = (2x+2)dx = 2(x+1)dx$, and $w = x+1$, so that $dw = dx$, and finally $2z^2 = w^2$, so that $z = w/\sqrt{2}$ and $dz = dw/\sqrt{2}$, or $dw = \sqrt{2}dz$.

3.

$$\int \frac{x}{x^2+4x+3}\,dx$$

Use partial fractions - decompose as

$$\frac{x}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

and we can find $A = -1/2$ and $B = 3/2$. Then

$$\int \frac{x}{(x+1)(x+3)}\,dx = \int \frac{A}{x+1} + \frac{B}{x+3}\,dx = \int \frac{-1/2}{x+1}\,dx + \int \frac{3/2}{x+3}\,dx$$
$$= -\frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x+3| + C$$

4.

$$\int x^2 \sin(2x)\,dx$$

Use integration by parts. Let $f(x) = 2x$ and $g'(x) = \sin(2x)$, so that $f'(x) = 2$ and $g(x) = \frac{-\cos(2x)}{2}$.

$$\int x^2 \sin(2x)\,dx = \frac{-x\cos(2x)}{2} + \int \cos(2x)\,dx$$
$$= \frac{-x\cos(2x)}{2} + \frac{\sin(2x)}{2} + C$$

5.

$$\int \ln(x)\,dx$$

Use integration by parts. Let $f(x) = \ln(x)$ and $g'(x) = 1$, so that $f'(x) = 1/x$ and $g(x) = x$. Then

$$\int \ln(x)\,dx = x\ln(x) - \int 1\,dx$$
$$= x\ln(x) - x + C$$

6.

$$\int \frac{x}{(2x+1)^{4/5}}\,dx$$

Here we can use either substitution or int. by parts. Let $u = 2x + 1$, then $du = 2dx$, so

$$\begin{aligned}
&= \int \frac{x}{(2x+1)^{4/5}}\, dx \\
&= \frac{1}{2}\int \frac{\frac{u-1}{2}}{u^{4/5}}\, du \\
&= \frac{1}{4}\int \frac{u-1}{u^{4/5}}\, du \\
&= \frac{1}{4}\int u^{1/5} - u^{-4/5}\, du \\
&= \frac{1}{4}\left[\frac{5}{6}u^{6/5} - 5u^{1/5}\right] + C \\
&= \frac{1}{4}\left[\frac{5}{6}(2x+1)^{6/5} - 5(2x+1)^{1/5}\right] + C
\end{aligned}$$

7.

$$\int \frac{1}{x(x-2)^3}\, dx$$

Use partial fractions - decompose as

$$\frac{1}{x(x-2)^3} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

You can find that $A = -1/8$, $B = 1/8$, $C = -1/4$, and $D = 1/2$. So we have

$$\begin{aligned}
\int \frac{1}{x(x-2)^3}\, dx &= \int \frac{-1}{8x} + \frac{1}{8(x-2)} + \frac{-1}{4(x-2)^2} + \frac{1}{2(x-2)^3}\, dx \\
&= \frac{-\ln|x|}{8} + \frac{\ln|x-2|}{8} + \frac{1}{4(x-2)} - \frac{1}{4(x-2)^2} + C
\end{aligned}$$

8.

$$\int \frac{6x+4}{(x-1)(x^2+2x+2)}\, dx$$

Use partial fractions - decompose as

$$\frac{6x+4}{(x-1)(x^2+2x+2)} = \frac{A}{x-1} + \frac{Bx+C}{(x+1)^2+1}$$

and we can find that $A = 2$, $B = -2$ and $C = 0$. So

$$\begin{aligned}
\int \frac{6x+4}{(x-1)(x^2+2x+2)}\,dx &= \int \frac{2}{x-1} - \frac{2x}{(x+1)^2+1}\,dx \\
&= 2\ln|x-1| - \int \frac{2(x+1)-2}{(x+1)^2+1}\,dx \\
&= 2\ln|x-1| - \int \frac{1}{u}\,du + \int \frac{2}{(x+1)^2+1}\,dx \\
&= 2\ln|x-1| - \ln|(x+1)^2+1| + 2\int \frac{1}{w^2+1}\,dw \\
&= 2\ln|x-1| - \ln|(x+1)^2+1| + 2\arctan(x+1) + C
\end{aligned}$$

where we let $u = (x+1)^2+1$, so that $du = 2(x+1)dx$ and $w = x+1$ so that $dw = dx$.