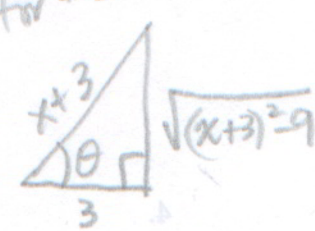
for #3:



Work-It-Out Day 6: Sections 7.2 and 7.3

Math 142 - Spring 2016

1.

$$\int \sqrt{27+9x^2}\,dx = \int 3\sqrt{3+x^2}\,dx$$

let $x=\sqrt{3}\tan\theta$
$\Rightarrow dx=\sqrt{3}\sec^2\theta\, d\theta$

$$= 3\int\sqrt{3+3\tan^2\theta}\cdot\sqrt{3}\sec^2\theta\, d\theta = 3\int\sqrt{3\sec^2\theta}\cdot\sqrt{3}\sec^2\theta\, d\theta$$

okay to use reduction formula for powers of secant

$$= 9\int\sec^3\theta\, d\theta = 9\left[\frac{\sec\theta\tan\theta}{2}+\frac{1}{2}\ln|\sec\theta\tan\theta|\right]+C$$

OR since: IBP

$$\int\sec^3\theta\, d\theta = \sec\theta\tan\theta - \int\sec\theta\tan^2\theta\, d\theta$$

$f=\sec\theta \Rightarrow f'=\sec\theta\tan\theta$
$g'=\sec^2\theta \Rightarrow g=\tan\theta$

$$= \sec\theta\tan\theta - \int\sec\theta(\sec^2\theta-1)\,d\theta = \sec\theta\tan\theta - \int\sec^3\theta\, d\theta + \int\sec\theta\, d\theta$$

So: $2\int\sec^3\theta\, d\theta = \sec\theta\tan\theta + \ln|\sec\theta+\tan\theta| + C$

2.

$$\int\frac{\cos^5(5x)}{\sin^3(5x)}\,dx$$

$u=\sin(5x)$
$du=5\cos(5x)\,dx$

$$\int\frac{\cos^4(5x)}{\sin^3(5x)}\cdot\cos(5x)\,dx = \int\frac{(1-\sin^2(5x))^2}{\sin^3(5x)}\cdot\cos(5x)\,dx$$

$$= \int\frac{(1-u^2)^2}{u^3}\cdot\frac{1}{5}du = \frac{1}{5}\int\frac{1-2u^2+u^4}{u^3}du = \frac{1}{5}\int u^{-3}-2u^{-1}+u\, du$$

$$= \frac{1}{5}\left[-\frac{1}{2}\sin^{-2}(5x) - 2\ln|\sin(5x)| + \frac{1}{2}\sin^2(5x)\right]+C$$

3.

$$\int\frac{1}{\sqrt{x^2+6x}}\,dx$$

$$\int\frac{1}{\sqrt{(x+3)^2-9}}\,dx = \int\frac{3\sec\theta\tan\theta}{\sqrt{9\sec^2\theta-9}}\,d\theta = \int\frac{3\sec\theta\tan\theta}{\sqrt{9\tan^2\theta}}\,d\theta = \int\sec\theta\, d\theta$$

let $x+3=3\sec\theta$
$\Rightarrow dx=3\sec\theta\tan\theta$

$$= \int\sec\theta\left(\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right)d\theta = \int\frac{\sec^2\theta+\sec\theta\tan\theta}{\sec\theta+\tan\theta}\,d\theta$$

$u=\sec\theta+\tan\theta$
$du=\sec\theta\tan\theta+\sec^2\theta\, d\theta$

$$= \int\frac{1}{u}du = \ln|\sec\theta+\tan\theta|+C = \ln\left|\frac{x+3}{3}+\frac{\sqrt{x^2+6x}}{3}\right|+C$$

4.

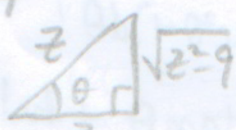
$$\int \sin^4(y)\,dy = \int \left(\frac{1-\cos(2y)}{2}\right)^2 dy$$

$$= \frac{1}{4}\int \left(1 - 2\cos(2y) + \cos^2(2y)\right) dy = \frac{1}{4}\left[y - \sin(2y) + \int \cos^2(2y)\,dy\right]$$

$$= \frac{1}{4}\left[y - \sin(2y) + \int \left(\frac{1+\cos(4y)}{2}\right) dy\right] = \frac{1}{4}\left[y - \sin(2y) + \frac{1}{2}\left[y + \frac{\sin(4y)}{4}\right]\right] + C$$

$$= \frac{3}{4}y - \frac{\sin(2y)}{4} + \frac{1}{8}\sin(4y) + C$$

5.



$$\int \frac{dx}{z^3\sqrt{z^2-9}}\,dz$$

let $z = 3\sec\theta$
$\Rightarrow dz = 3\sec\theta\tan\theta\,d\theta$

$$= \frac{1}{27}\int \frac{3\sec\theta\tan\theta}{\sec^3\theta\sqrt{9\sec^2\theta - 9}}\,d\theta = \frac{1}{27}\int \frac{\cancel{3\sec\theta\tan\theta}}{3\sec^3\theta\,\cancel{\tan\theta}}\,d\theta = \frac{1}{27}\int \frac{1}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{27}\int \cos^2\theta\,d\theta = \frac{1}{27}\int \left(\frac{1+\cos(2\theta)}{2}\right) d\theta = \frac{1}{27}\left[\frac{\theta}{2} + \frac{1}{4}\underbrace{\sin(2\theta)}_{2\sin\theta\cos\theta}\right] + C$$

$$= \frac{1}{54}\operatorname{arcsec}\left(\frac{z}{3}\right) + \frac{1}{54}\left(\frac{\sqrt{z^2-9}}{z}\right)\left(\frac{3}{z}\right) + C$$

6.

$$\int \frac{z^3}{\sqrt{9-z^2}}\,dz$$

let $u = 9 - z^2$
$\Rightarrow du = -2z\,dz$
and $z^2 = 9 - u$

$$\Rightarrow \int \frac{(9-u)\cdot(-\frac{1}{2})\,du}{u^{1/2}} = -\frac{1}{2}\int 9u^{-1/2} - u^{1/2}\,du$$

$$= -\frac{1}{2}\left[\frac{9u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2}\right] + C = -\frac{1}{2}\left[18(9-z)^{1/2} - \frac{2}{3}(9-z)^{3/2}\right] + C$$