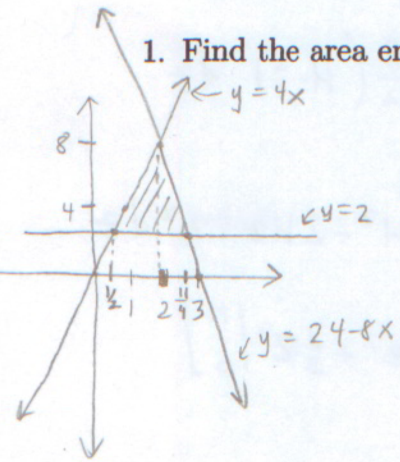
Work-It-Out Day 7: Sections 6.1,6.2 and 6.3
Math 142 - Spring 2016

1. Find the area enclosed by the graphs of $y = 4x$, $y = 24 - 8x$, $y = 2$.



$$\text{Ⓐ Area} = \int_2^8 \left(\frac{24-y}{8}\right) - \left(\frac{y}{4}\right) dy = 3y - \frac{y^2}{16} - \frac{y^2}{8}\Big|_2^8 = 3y - \frac{3y^2}{16}\Big|_2^8$$

$$= \left(24 - \frac{3\cdot 64}{16}\right) - \left(6 - \frac{12}{16}\right)$$

$$= 18 - \frac{180}{16} = 18 - 11\tfrac{1}{4}$$

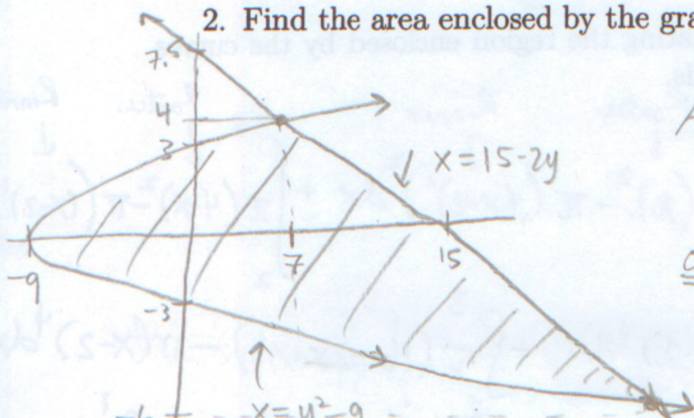
$$= 6\tfrac{3}{4}.$$

OR

$$\text{Ⓑ Area} = \int_{1/2}^{2} (4x-2)\,dx + \int_2^{11/4} (24-8x) - 2\,dx$$

2. Find the area enclosed by the graphs of $x = y^2 - 9$, $x = 15 - 2y$.

$$y^2 - 9 = 15 - 2y$$
$$y^2 + 2y - 24 = 0$$
$$(y+6)(y-4) = 0$$



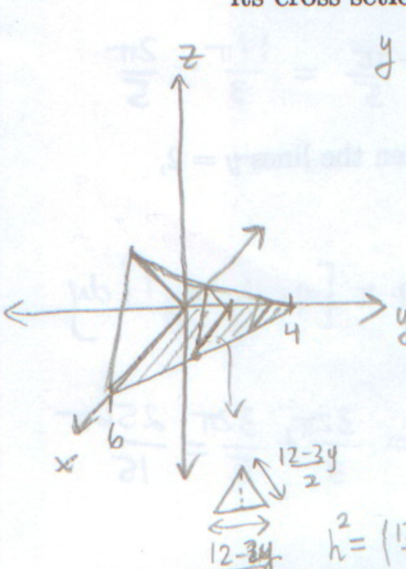
$$\text{Area} = \int_{-6}^{4} (15-2y) - (y^2-9)\,dy$$

OR

$$\text{Area} = \int_{-9}^{7} \sqrt{x+9} - (-\sqrt{x+9})\,dx + \int_7^{27} \left(\frac{15-x}{2}\right) - (-\sqrt{x+9})\,dx$$

3. The base of a solid is a triangle bounded by the axes and the line $2x + 3y = 12$, and its cross setions perpindicular to the y-axis are equilateral triangles. Find it's volume.

$$y = \frac{12-2x}{3} = 4 - \tfrac{2}{3}x$$



$$\text{Volume} = \int_0^4 \frac{1}{2}\left(\frac{12-3y}{2}\right)\left(\sqrt{\frac{3}{16}}(12-3y)\right) dy$$

$$= \frac{\sqrt{3}}{16}\int_0^4 (12-3y)^2\,dy$$

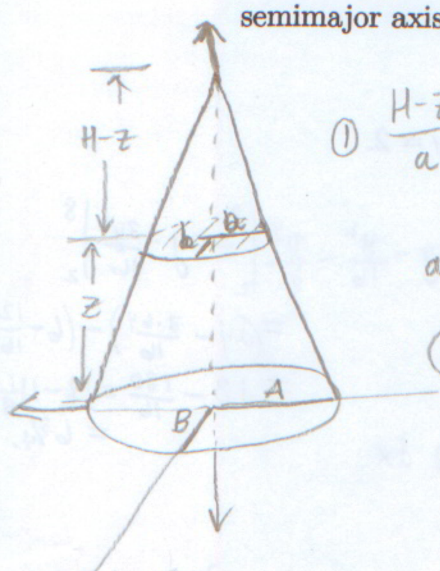
$$= \frac{\sqrt{3}}{16}\int_0^4 12^2 - 72y + 9y^2\,dy$$

$$h^2 = \left(\frac{12-3y}{2}\right)^2 - \left(\frac{12-3y}{4}\right)^2$$

$$h^2 = \frac{1}{16}\left(3(12-3y)^2\right)$$

$$= \frac{\sqrt{3}}{16}\left[144y - 36y^2 + 3y^3\Big|_0^4\right] =$$

4. Find an equation for the volume of a cone of height H whose base is an ellipse with semimajor axis A and semiminor axis B. (Note the area of an ellipse is πab.



① $\frac{H-z}{a} = \frac{H}{A}$

$\Downarrow$

$a = \frac{A}{H}(H-z)$

(B) $\frac{H-z}{b} = \frac{H}{B}$

$\Downarrow$

$b = \frac{B}{H}(H-z)$

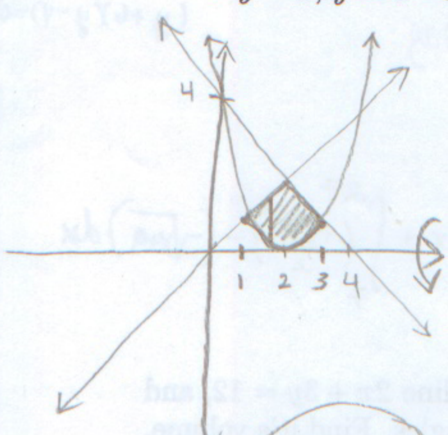
$$\text{Volume} = \int_0^H \frac{\pi AB}{H^2}(H-z)^2\,dz$$

$$= \frac{\pi AB}{H^2}\int_0^H H^2 - 2Hz + z^2\,dz$$

$$= \frac{\pi AB}{H^2}\left[H^2 z - Hz^2 + \tfrac{1}{3}z^3\Big|_0^H\right]$$

$$= \frac{\pi ABH}{3}$$

5. Find the volume of the solid obtained by rotating the region enclosed by the curves $y = x$, $y = 4 - x$, $y = (x-2)^2$ about the x-axis.



$$\text{Volume} = \int_1^2 \pi\,\underbrace{(x)}_{R_{outer}}{}^2 - \pi\,\underbrace{((x-2)^2)}_{R_{inner}}{}^2\,dx + \int_2^3 \pi\,\underbrace{(4-x)}_{R_{outer}}{}^2 - \pi\,\underbrace{((x-2)^2)}_{R_{inner}}{}^2\,dx$$

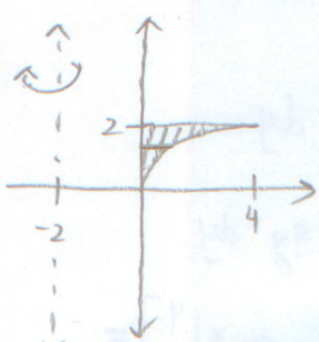
$$= \int_1^2 \pi x^2 - \pi(x-2)^4\,dx + \int_2^3 \pi(16-8x+x^2) - \pi(x-2)^4\,dx$$

$$= \frac{\pi}{3}x^3\Big|_1^2 - \pi\int_{-1}^0 u^4\,du + \pi\left[16x - 4x^2 + \tfrac{1}{3}x^3\Big|_2^3\right] - \pi\int_0^1 u^4\,du$$

$u = x-2$
$du = dx$

$$= \frac{7\pi}{3} - \pi\left(-\frac{(-1)^5}{5}\right) + \pi\left[21 - 16 - \frac{8}{3}\right] - \frac{\pi}{5} = \frac{14\pi}{3} - \frac{2\pi}{5}$$

6. Find the volume of the solid obtained by rotating the region between the lines $y = 2$, $x = 0$, and $y = \sqrt{x}$ about the line $x = -2$.



$$V = \int_0^2 -\pi(2)^2 + \pi(2+y^2)^2\,dy = \int_0^2 -4\pi + \pi\left[4 + 4y^2 + y^4\right]dy$$

$$= \pi\int_0^2 4y^2 + y^4\,dy = \pi\left(\frac{4}{3}y^3 + \frac{1}{5}y^5\right)\Big|_0^2 = \frac{32\pi}{3} + \frac{32\pi}{5} = \frac{256\pi}{15}$$