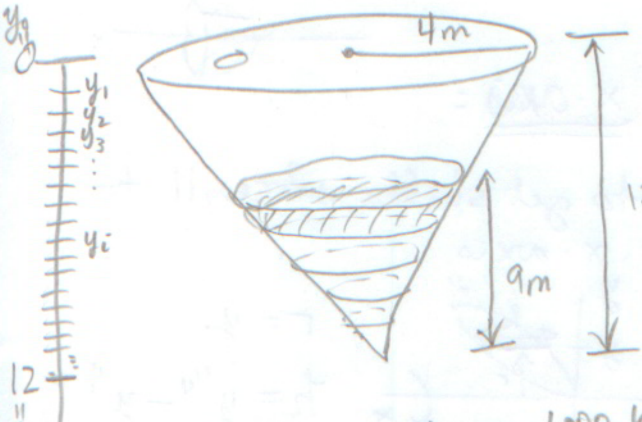
Example of Work:



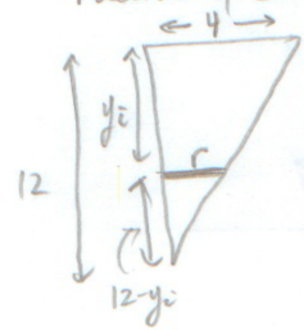
density of $H_2O = 1000\ kg/m^3$

Work required to pump all water to tip of cone?

On a single slice – approximated by a cylinder,

$$V_i \approx 2\pi \underline{r}\, \Delta y$$

radius of slice i?



$$\Rightarrow \frac{4}{r} = \frac{12}{12-y_i}$$

$$\Rightarrow \boxed{r = \frac{1}{4}\left(1-\frac{y_i}{12}\right)}$$

volume of ith slice:

$$V_i \approx 2\pi\left(\frac{1}{4}\left(1-\frac{y_i}{12}\right)\right)\Delta y$$

Work on ith slice: $$W_i \approx \underbrace{\left(1000\,\frac{kg}{m^3}\cdot 2\pi\left(\frac{1}{4}\left(1-\frac{y_i}{12}\right)\right)\Delta y\right)}_{\text{mass}}\cdot g \cdot \underset{\uparrow\ \text{displacement}}{y_i}$$

$\Rightarrow$ total work $$W \approx \sum_{i=1}^{N} 1000\cdot\frac{2\pi}{4}g\left(1-\frac{y_i}{12}\right)y_i\,\Delta y$$

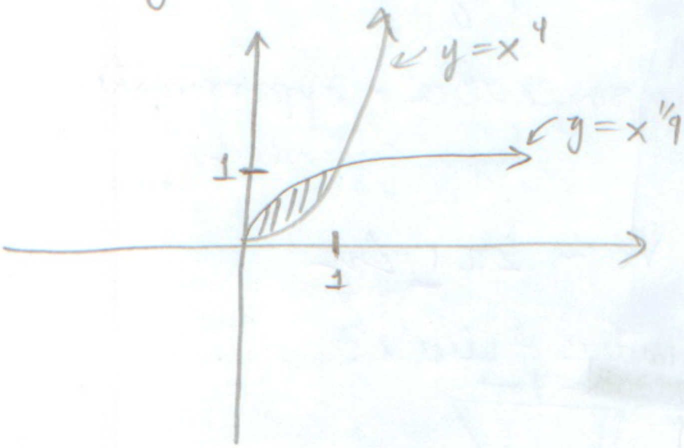
Limit as $N\to\infty$ $$\Rightarrow \quad W = \int_3^9 500\pi g\left(1-\frac{y}{12}\right)y\,dy$$

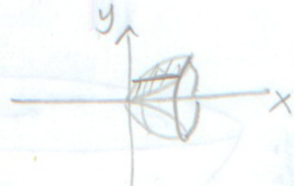
$\uparrow$ b/c tank is not full + water begins at $y=3$.

$$W = 500\pi g\left[\frac{y^2}{2}-\frac{y^3}{36}\Big|_3^9\right] = \ldots$$

Example of Shell Method:

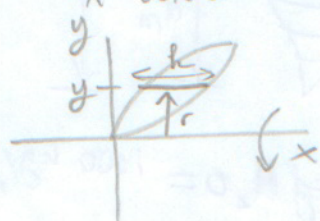
region bdd by: $y = x^4$, $y = x^{1/4}$ rotated about the





(A) x-axis:

to get shells slice ∥ to x-axis



$r = y$

$h = y^{1/4} - y^4$

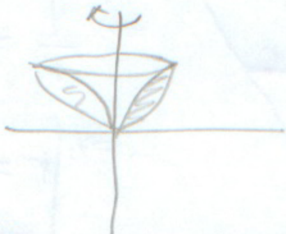
$$V = \int_0^1 2\pi y\,(y^{1/4} - y^4)\,dy$$

$$= 2\pi \int_0^1 y^{5/4} - y^5\,dy$$

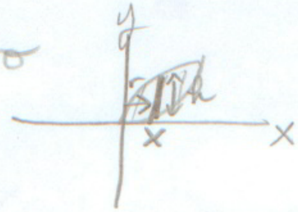
$$= 2\pi \left[\frac{4y^{9/4}}{9} - \frac{y^6}{6} \Big|_0^1 \right]$$

$$= 2\pi \left[\frac{4}{9} - \frac{1}{6} \right].$$

(B) y-axis



to get shells here slice ∥ to y-axis



$r = x$

$h = x^{1/4} - x^4$

$$V = \int_0^1 2\pi x\,(x^{1/4} - x^4)\,dx$$

$$= 2\pi \int_0^1 x^{5/4} - x^5\,dx$$

etc...