

Solutions.

Work-It-Out Day 9: Sections 7.7, 7.9 Math 142 - Spring 2016

1. Evaluate $\int_{-\infty}^{\infty} x^3 e^{-x^4} dx$ if it converges, or state that it diverges.

$$\begin{aligned}
 &= \lim_{R \rightarrow \infty} \int_0^R x^3 e^{-x^4} dx + \lim_{R \rightarrow -\infty} \int_R^0 x^3 e^{-x^4} dx \quad \text{let } u = -x^4 \Rightarrow du = -4x^3 dx \\
 &= -\frac{1}{4} \lim_{R \rightarrow \infty} \int_0^R e^u du - \frac{1}{4} \lim_{R \rightarrow -\infty} \int_R^0 e^u du \\
 &= -\frac{1}{4} \lim_{R \rightarrow \infty} \left(\frac{e^{-x^4}}{-4} \Big|_0^R \right) - \frac{1}{4} \lim_{R \rightarrow -\infty} \left(\frac{e^{-x^4}}{-4} \Big|_R^0 \right) = -\frac{1}{4} \lim_{R \rightarrow \infty} (e^{-R^4} - 1) - \frac{1}{4} \lim_{R \rightarrow -\infty} (1 - e^{-R^4}) \\
 &= +\frac{1}{4} - \frac{1}{4} = \underline{0} \quad \leftarrow \text{converges to zero!}
 \end{aligned}$$

2. Evaluate $\int_3^{\infty} \frac{1}{x+2} - \frac{1}{x-2} dx$ if it converges, or state that it diverges.

bottom limit should be 3.

$$\lim_{R \rightarrow \infty} \left(\int_3^R \frac{1}{x+2} - \frac{1}{x-2} dx \right) = \lim_{R \rightarrow \infty} \left(\int_5^{R+2} \frac{1}{u} du - \int_{+1}^{R-2} \frac{1}{w} dw \right)$$

used:

$$\begin{aligned}
 u &= x+2 \\
 \Rightarrow du &= dx \\
 \text{and} \\
 w &= x-2 \\
 \Rightarrow dw &= dx
 \end{aligned}$$

$$= \lim_{R \rightarrow \infty} \left(\ln|u| \Big|_5^{R+2} - \ln|w| \Big|_1^{R-2} \right) = \lim_{R \rightarrow \infty} \left(\ln|R+2| - \ln|R-2| - \ln|5| + \ln|1| \right)$$

$$= \lim_{R \rightarrow \infty} \left(\ln \left| \frac{R+2}{R-2} \right| - \ln|5| \right) = \ln|1| - \ln|5| = \underline{-\ln|5|} \quad \leftarrow \text{converges to } -\ln|5|$$

3. Evaluate $\int_{-2}^1 \frac{3}{x^{2/5}} dx$ if it converges, or state that it diverges.

$$= \int_{-2}^0 \frac{3}{x^{2/5}} dx + \int_0^1 \frac{3}{x^{2/5}} dx = \lim_{R \rightarrow 0^-} \left(\int_{-2}^R \frac{3}{x^{2/5}} dx \right) + \lim_{R \rightarrow 0^+} \left(\int_R^1 \frac{3}{x^{2/5}} dx \right)$$

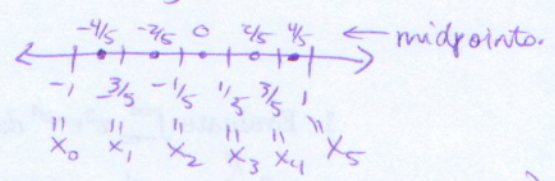
$$= \lim_{R \rightarrow 0^-} \left(\frac{3x^{3/5}}{3/5} \Big|_{-2}^R \right) + \lim_{R \rightarrow 0^+} \left(\frac{3x^{3/5}}{3/5} \Big|_R^1 \right) = \lim_{R \rightarrow 0^-} \left(5R^{3/5} - 5(-2)^{3/5} \right) + \lim_{R \rightarrow 0^+} \left(5 - 5R^{3/5} \right)$$

$$= -5(-2)^{3/5} + 5 = -5\sqrt[5]{-8} + 5 = \underline{5\sqrt[5]{8} + 5}$$

converges to $5\sqrt[5]{8} + 5$

4. Calculate T_N and M_N for the value of $N = 5$. $\Rightarrow \Delta x = \frac{(1) - (-1)}{5} = \frac{2}{5}$

$$\int_{-1}^1 3e^{x^4} dx$$



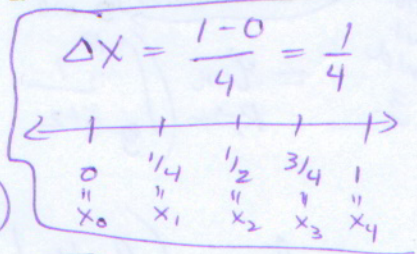
$$\int_{-1}^1 3e^{x^4} dx \approx T_5 = \frac{2}{5} \left[3e^{(-1)^4} + 2(3e^{(-3/5)^4}) + 2(3e^{(-1/5)^4}) + 2(3e^{(1/5)^4}) + 2(3e^{(3/5)^4}) + 3e^{1^4} \right]$$

$$\approx \frac{1}{5} \left[3e + 6e^{3^4/5^4} + 6e^{1^4/5^4} + 6e^{1^4/5^4} + 6e^{3^4/5^4} + 3e \right]$$

$$\int_{-1}^1 3e^{x^4} dx \approx M_5 = \frac{2}{5} \left[3e^{(-4/5)^4} + 3e^{(-2/5)^4} + 3e^{(0)^4} + 3e^{(2/5)^4} + 3e^{(4/5)^4} \right]$$

5. Calculate S_N given by Simpson's Rule for the value of $N = 4$.

$$\int_0^1 e^{-x^4} dx$$



$$\int_0^1 e^{-x^4} dx \approx S_4 = \frac{1}{3} \left[f(0) + 4f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + 4f\left(\frac{3}{4}\right) + f(1) \right]$$

$$\approx \frac{1}{12} \left[e^0 + 4e^{-(1/4)^4} + 2e^{-(1/2)^4} + 4e^{-(3/4)^4} + e^{-1} \right]$$

6. Use the error bound formula to find the smallest value of N for which $\text{Error}(S_N) \leq 10^{-7}$.

$$f(x) = x^{3/5} \Rightarrow f'(x) = \frac{3}{5}x^{-2/5} \Rightarrow f''(x) = -\frac{6}{25}x^{-7/5}$$

$$f'''(x) = \frac{42}{125}x^{-17/5} \Rightarrow f^{(4)}(x) = -\frac{504}{625}x^{-17/5}$$

↑ decreasing function on (2,5)

$$\text{error}(S_N) \leq \frac{K_4(b-a)^5}{180 \cdot N^4}$$

-let $K_2 = \frac{504}{625} \cdot 2^{-17/5}$

$$\Rightarrow \text{error}(S_N) \leq \frac{504}{625} (2)^{-17/5} \cdot (5-2)^5 \leq 10^{-7} \Rightarrow \frac{504 \cdot 2^{-17/5} \cdot 3^5}{180 \cdot 10^{-7}} \leq N^4$$

$$\Rightarrow 1,031,293.55 \leq N^4 \Rightarrow \underline{31.87} \leq N \Rightarrow N \text{ must be at least } 32 \text{ to guarantee accuracy to } 10^{-7}$$