Solutions.

Work-It-Out Day 9: Sections 7.7, 7.9
Math 142 - Spring 2016

1. Evaluate $\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx$ if it converges, or state that it diverges.

let $u = -x^4 \Rightarrow du = -4x^3dx$

$$= \lim_{R\to\infty}\int_0^R x^3 e^{-x^4}dx + \lim_{R\to-\infty}\int_R^0 x^3 e^{-x^4}dx = -\frac{1}{4}\lim_{R\to\infty}\int_0^R e^u du - \frac{1}{4}\lim_{R\to-\infty}\int_R^0 e^u du$$

$$= -\frac{1}{4}\lim_{R\to\infty}\left(e^{-x^4}\Big|_0^R\right) - \frac{1}{4}\lim_{R\to-\infty}\left(e^{-x^4}\Big|_R^0\right) = -\frac{1}{4}\lim_{R\to\infty}\left(e^{-R^4}-1\right) - \frac{1}{4}\lim_{R\to-\infty}\left(1-e^{-R^4}\right)$$

$$= +\frac{1}{4} - \frac{1}{4} = \underline{0} \leftarrow \boxed{\text{so converges to zero!}}$$

bottom limit should be 3.

2. Evaluate $\int_{3}^{\infty} \frac{1}{x+2} - \frac{1}{x-2}\,dx$ if it converges, or state that it diverges.

$$= \lim_{R\to\infty}\left(\int_3^R \frac{1}{x+2} - \frac{1}{x-2}\,dx\right) = \lim_{R\to\infty}\left(\int_5^{R+2}\frac{1}{u}du - \int_1^{R-2}\frac{1}{w}dw\right)$$

used: $u = x+2 \Rightarrow du = dx$ and $w = x-2 \Rightarrow dw = dx$

$$= \lim_{R\to\infty}\left(\ln|u|\Big|_5^{R+2} - \ln|w|\Big|_1^{R-2}\right) = \lim_{R\to\infty}\left(\ln|R+2| - \ln|R-2| - \ln|5| + \ln|1|\right)$$

$$= \lim_{R\to\infty}\left(\ln\left|\frac{R+2}{R-2}\right| - \ln|5|\right) = \ln|1| - \ln|5| = \underline{-\ln|5|}. \quad \boxed{\text{converges to } -\ln|5|}$$

3. Evalute $\int_{-2}^{1} \frac{3}{x^{2/5}}\,dx$ if it converges, or state that it diverges.

$$= \int_{-2}^0 \frac{3}{x^{2/5}}dx + \int_0^1 \frac{3}{x^{2/5}}dx = \lim_{R\to0^-}\left(\int_{-2}^R \frac{3}{x^{2/5}}dx\right) + \lim_{R\to0^+}\left(\int_R^1 \frac{3}{x^{2/5}}dx\right)$$

$$= \lim_{R\to0^-}\left(\frac{3x^{3/5}}{3/5}\Big|_{-2}^R\right) + \lim_{R\to0^+}\left(\frac{3x^{3/5}}{3/5}\Big|_R^1\right) = \lim_{R\to0^-}\left(5R^{3/5} - 5(-2)^{3/5}\right) + \lim_{R\to0^+}\left(5 - 5R^{3/5}\right)$$

$$= -5(-2)^{3/5} + 5 = -5\sqrt[5]{-8} + 5 = \underline{5\sqrt[5]{8} + 5}. \quad \boxed{\text{Converges to } 5\sqrt[5]{8}+5}$$

Solutions

4. Calculate T_N and M_N for the value of $N = 5$. $\Rightarrow \Delta x = \frac{(1)-(-1)}{5} = \frac{2}{5}$

$$\int_{-1}^{1} 3e^{x^4}\,dx$$

$-4/5 \quad -2/5 \quad 0 \quad 2/5 \quad 4/5 \leftarrow$ midpoints.

$-1 = x_0, \; -3/5 = x_1, \; -1/5 = x_2, \; 1/5 = x_3, \; 3/5 = x_4, \; 1 = x_5$

$$\int_{-1}^{1} 3e^{x^4}dx \approx T_5 = \frac{\frac{2}{5}}{2}\left[3e^{(-1)^4} + 2(3e^{(-3/5)^4}) + 2(3e^{(-1/5)^4}) + 2(3e^{(1/5)^4}) + 2(3e^{(3/5)^4}) + 3e^{1^4}\right]$$

$$\approx \frac{1}{5}\left[3e + 6e^{3^4/5^4} + 6e^{1/5^4} + 6e^{1/5^4} + 6e^{3^4/5^4} + 3e\right]$$

$$\int_{-1}^{1} 3e^{x^4}dx \approx M_5 = \frac{2}{5}\left[3e^{(-4/5)^4} + 3e^{(-2/5)^4} + 3e^{(0)^4} + 3e^{(2/5)^4} + 3e^{(4/5)^4}\right]$$

5. Calculate S_N given by Simpson's Rule for the value of $N = 4$.

$$\int_0^1 e^{-x^4}\,dx$$

$\Delta x = \frac{1-0}{4} = \frac{1}{4}$

$0 = x_0, \; 1/4 = x_1, \; 1/2 = x_2, \; 3/4 = x_3, \; 1 = x_4$

$$\int_0^1 e^{-x^4}dx \approx S_4 = \frac{1/4}{3}\left[f(0) + 4f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 4f(3/4) + f(1)\right]$$

$$\approx \frac{1}{12}\left[e^0 + 4e^{-(\frac{1}{4})^4} + 2e^{-(\frac{1}{2})^4} + 4e^{-(3/4)^4} + e^{-1}\right]$$

6. Use the error bound formula to find the smallest value of N for which Error(S_N) $\leq 10^{-7}$.

$$\int_2^5 x^{3/5}\,dx$$

$f(x) = x^{3/5} \Rightarrow f'(x) = 3/5\, x^{-2/5} \Rightarrow f''(x) = -\frac{6}{25}x^{-7/5}$

$f'''(x) = \frac{42}{125}x^{-12/5} \Rightarrow f^{(4)}(x) = \underbrace{\frac{-504}{625}x^{-17/5}}_{\text{decreasing function on } [2,5]}$

$\boxed{\text{error}(S_N) \leq \frac{K_4(b-a)^5}{180 \cdot N^4}}$

$-$ let $K_2 = \frac{504}{625}\cdot 2^{-17/5}$

$\Rightarrow \text{error}(S_N) \leq \frac{\frac{504}{625}(2)^{-17/5}\cdot(5-2)^5}{180\cdot N^4} \overset{\text{want}}{\leq} 10^{-7} \Rightarrow \frac{\frac{504}{625}\cdot 2^{-17/5}\cdot 3^5}{180\cdot 10^{-7}} \leq N^4$

$\Rightarrow 1{,}031{,}293.55 \leq N^4 \Rightarrow \underline{31.87 \leq N} \Rightarrow$ N must be at least 32 to guarantee accuracy to 10^{-7}.