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October 29, 2013

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- Locusts only reproduce during the adult stage of their life - the average female adult produces 1000 eggs before dying.
- Adults die soon after reproduction.

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- Females who reach age of 2 years survive an additional year with probability 0.95 and reproduce with the probability of 0.42.
- There is a 0.6 chance that a calf survives to be a yearling.
- There is a 0.7 chance that a yearling survives to adulthood.

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We clearly see the total female population is growing each year. While the trend in each individual class is not so clear.

General Leslie Matrix Definitions

In general a Leslie matrix model has the form $\vec{x}(t+1) = A\vec{x}(t)$, where

$$A = \begin{bmatrix} F_1 & F_2 & F_3 & \dots & F_n \\ S_1 & 0 & 0 & \dots & 0 \\ 0 & S_2 & 0 & \dots & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots S_{n-1} & S_n \end{bmatrix}$$

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where F_i is the average number of female offspring born to a female individual per time step in class i , and S_i is the percent of individual in class i that survive to class $i+1$ in each time step , if $i < n$. S_n is the percent of individuals in the final class n that survive to the next year and remain in the same class.

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- Leslie matrices ALWAYS have a unique positive eigenvalue.