#18: $$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C \qquad \text{(substituted } x = at\text{)}$$

$$\int \frac{dx}{(x+a)^2} = -\frac{1}{x+a} + C \qquad \text{(substituted } x = t-a\text{)}$$

$$\int \frac{dx}{x+a} = \ln(x+a) + C \qquad \text{(-"- ; assuming } x+a>0\text{)}$$

$$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln(x^2+a^2) + C \qquad \text{(substituted } x^2 = t\text{, or else } x^2+a^2 = t\text{)}$$

$$\int \frac{x+a}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2) + \arctan\frac{x}{a} + C \qquad \text{(taken apart integrand as } \frac{x}{x^2+a^2} + a\cdot\frac{1}{x^2+a^2}\text{)}$$

$$\int \frac{\ln x}{x}\,dx = \frac{1}{2}(\ln x)^2 + C \qquad \text{(substituted } \ln x = t\text{)}$$

$$\int \frac{1}{x\ln x}\,dx = \ln(\ln x) + C \qquad \text{(-"-)}$$

$$\int \ln x\,dx = x\ln x - x + C \qquad \left(\text{because } \frac{d}{dx}(x\ln x) = \ln x - 1\right)$$

#19: $$\int \frac{x+1}{x^2+2x+1}\,dx = \int \frac{dx}{x+1} = \ln(x+1) + C \qquad \text{(for } x>-1\text{)}$$

$$\int \frac{1}{x^2+2x+1}\,dx = \int \frac{dx}{(x+1)^2} = -\frac{1}{x+1} + C \qquad \text{(for } x\neq -1\text{)}$$

So $$\int \frac{x}{x^2+2x+1}\,dx = \ln(x+1) + \frac{1}{x+1} + C$$

$$\int \frac{x+1}{x^2+2x+2}\,dx = \int \frac{x+1}{(x+1)^2+1}\,dx \underset{x+1=t}{=} \int \frac{t}{t^2+1}\,dt = \frac{1}{2}\ln(t^2+1) + C$$

$$= \frac{1}{2}\ln\left((x+1)^2+1\right) + C$$

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1} = \int \frac{dt}{t^2+1} = \arctan t + C = \arctan(x+1) + C$$