$$\int \frac{x}{x^2+2x+2}\,dx = \int \left(\frac{x+1}{(x+1)^2+1} - \frac{1}{(x+1)^2+1} \right) dx =$$

$$= \underline{\tfrac{1}{2}\ln\left((x+1)^2+1\right) - \arctan(x+1) + C}$$

and the big message here is that you take the integral apart in a way that can only be motivated by following the standard types studied in #19.

#20: Again heading for the standard types, we first complete the square in the denominator:

$$\frac{x+1}{x^2+x+1} = \frac{x+1}{(x+\frac{1}{2})^2+\frac{3}{4}}$$

Having thus a substitution $x+\frac{1}{2}=t$ in mind, we separate the numerator $x+1$ into $x+1 = (x+\frac{1}{2})+\frac{1}{2} = t+\frac{1}{2}$

$$\int_0^1 \frac{x+1}{x^2+x+1}\,dx \underset{x+\frac{1}{2}=t}{=} \int_{1/2}^{3/2} \frac{t+\frac{1}{2}}{t^2+\frac{3}{4}}\,dt =$$

$$\int_{1/2}^{3/2} \frac{t}{t^2+(\frac{\sqrt{3}}{2})^2}\,dt + \int_{1/2}^{3/2} \frac{1/2}{t^2+(\frac{\sqrt{3}}{2})^2}\,dt = \left[\frac{1}{2}\ln\left(t^2+\frac{3}{4}\right)\right]_{1/2}^{3/2} +$$

$$+ \frac{1}{2}\left[\frac{2}{\sqrt{3}}\arctan\frac{2t}{\sqrt{3}}\right]_{1/2}^{3/2} = \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1 + \frac{1}{\sqrt{3}}\left(\arctan\sqrt{3} - \arctan\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{2}\ln 3 + \frac{\sqrt{3}}{3}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \underline{\underline{\frac{1}{2}\ln 3 + \frac{\sqrt{3}\,\pi}{18}}} \approx 0.8516$$