#21:

$$\frac{1}{x} - \frac{1}{x+1} = \frac{x+1-x}{x(x+1)} = \frac{1}{x(x+1)}$$

$$\int \frac{dx}{x(x+1)} = \int \frac{dx}{x} - \int \frac{dx}{x+1} = \ln x - \ln(x+1) + C \ (\text{for } x > 0)$$

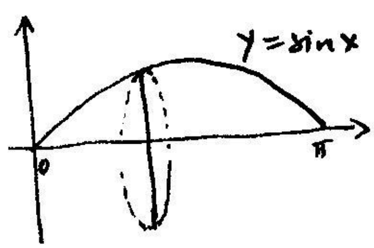
$$= \ln \frac{x}{x+1} + C$$

$$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{x(x+3)}$$

$$\int \frac{dx}{x(x+3)} = \frac{1}{3}\left(\int \frac{dx}{x} - \int \frac{dx}{x+3}\right) = \frac{1}{3}\left(\ln x - \ln(x+3)\right) + C$$

$$= \frac{1}{3} \ln \frac{x}{x+3} + C$$

#22:



$$\text{volume} = \int_0^{\pi} \pi \sin^2 x \, dx =$$

$$= \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx = \pi \left[\frac{x}{2} - \frac{\sin 2x}{4} \right]_0^{\pi}$$

$$= \pi \left(\frac{\pi}{2} - 0 \right) = \underline{\underline{\frac{\pi^2}{2}}}$$

#23:

arclength formula $\int \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

$$\text{length} = \int_0^{\pi} \underbrace{\sqrt{1 + \cos^2 x}}_{f(x)} \, dx .$$

A good numerical attempt (without much work) would be Simpson, say, with nodes $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$

$$h = \frac{\pi}{6}$$

$$L \approx \frac{1}{3} \cdot \frac{\pi}{6} \left(f(0) + 4f(\tfrac{\pi}{6}) + 2f(\tfrac{\pi}{3}) + 4f(\tfrac{\pi}{2}) + 2f(\tfrac{2\pi}{3}) + 4f(\tfrac{5\pi}{6}) + f(\pi) \right)$$

$$= \frac{\pi}{18} \left(\sqrt{2} + 2\sqrt{7} + \sqrt{5} + 4 + \sqrt{5} + 2\sqrt{7} + \sqrt{2} \right) = \frac{\pi}{18}\left(4 + 2\sqrt{2} + 2\sqrt{5} + 4\sqrt{7}\right) \approx \cancel{3.6748} \ 3.8194$$

precise numerics gives 3.8202